

1. (R4) Simplify: $(x^2 - 3x + 1) - (2x - 5)$ Simplifying $-(2x - 5)$ works as if you were multiplying $(2x - 5)$ by -1 , so you must use the distributive law. The sign is changed for *both* terms.

$$(x^2 - 3x + 1) - (2x - 5) = x^2 - 3x + 1 - 2x + 5 = x^2 - 5x + 6 \quad \text{Answer: (d) } x^2 - 5x + 6$$

2. (R4 #84) Expand and simplify $(2x + 3y)^2$. Remember, you must **not** be tempted to say that the answer is $4x^2 + 9y^2$. If you think you remember that as a "shortcut" you need to completely erase it from your memory.

It is safest to write this as a product $(2x + 3y)(2x + 3y)$ and then use the distributive law.

$$(2x + 3y)(2x + 3y) = 2x(2x + 3y) + 3y(2x + 3y) = 4x^2 + 6xy + 6yx + 9y^2 = 4x^2 + 12xy + 9y^2$$

$$\text{Answer: (c) } 4x^2 + 12xy + 9y^2$$

3. (R5 122) Factor completely: $4(x + 5)^3(x - 1)^2 + 2(x + 5)^4(x - 1)$

Think of this as two pieces: $4(x + 5)^3(x - 1)^2$ and $2(x + 5)^4(x - 1)$. You need to find any terms in common, and factor out as much as possible. Each one has a 2 as a factor, a term involving $x + 5$, and a term involving $x - 1$. To help understand the problem, you might rewrite the two different terms this way:

$2 \cdot 2(x + 5)^3(x - 1)(x - 1)$ and $2(x + 5)^3(x + 5)(x - 1)$. Then you can see that the factors that belong to both pieces are 2, $(x + 5)^3$, and $(x - 1)$.

$$4(x + 5)^3(x - 1)^2 + 2(x + 5)^4(x - 1) = 2(x + 5)^3(x - 1)[2(x - 1) + (x + 5)] = 2(x + 5)^3(x - 1)[2x - 2 + x + 5] = 2(x + 5)^3(x - 1)(3x + 3) = 6(x + 5)^3(x - 1)(x + 1) \quad \text{Answer: (c) } 6(x + 5)^3(x - 1)(x + 1)$$

4. (R6 10) When $5x^4 - x^2 + x - 2$ is divided by $x^2 + 2$ we get

$$\begin{array}{r} x^2 \quad +2 \quad \overline{) \begin{array}{r} 5x^4 \quad -x^2 \quad +x \quad -2 \\ 5x^4 \quad \quad +10x^2 \quad \quad \quad \\ \hline \quad \quad -11x^2 \quad +x \quad -2 \\ \quad \quad -11x^2 \quad \quad \quad -22 \\ \hline \quad \quad \quad \quad \quad \quad \quad x \quad +20 \end{array}} \end{array}$$

Answer: (b) quotient: $5x^2 - 11$; remainder: $x + 20$

5. (R7) Simplify and factor: $\frac{4 + \frac{1}{x^2}}{25 - \frac{1}{x^2}}$. Multiply the numerator and denominator by x^2 to simplify

$$\frac{1}{x^2}.$$

$$\frac{x^2(4 + \frac{1}{x^2})}{x^2(25 - \frac{1}{x^2})} = \frac{4x^2 + x^2 \cdot \frac{1}{x^2}}{25x^2 - x^2 \cdot \frac{1}{x^2}} = \frac{4x^2 + 1}{25x^2 - 1} = \frac{4x^2 + 1}{(5x - 1)(5x + 1)} \quad \text{Answer: (e)}$$

6. (R7 66) After simplifying $\frac{2}{(x + 2)^2(x - 1)} - \frac{6}{(x + 2)(x - 1)^2}$, the numerator is ??? Answer: (d) $-4x - 14$

You first need to find a common denominator. $\frac{2}{(x + 2)^2(x - 1)} - \frac{6}{(x + 2)(x - 1)^2}$

$$= \frac{2(x - 1)}{(x + 2)^2(x - 1)^2} + \frac{(-6)(x + 2)}{(x + 2)^2(x - 1)^2} = \frac{2x - 2 - 6x - 12}{(x + 2)^2(x - 1)^2} = \frac{-4x - 14}{(x + 2)^2(x - 1)^2}$$

7. (R7 72) After simplifying, the numerator of $\frac{1}{h} \left[\frac{1}{(x + h)^2} - \frac{1}{x^2} \right]$ is ??? Answer: (a) $-2x - h$

You need to find a common denominator in order to add the terms inside the brackets.

$$\begin{aligned} \frac{1}{h} \left[\frac{1}{(x + h)^2} - \frac{1}{x^2} \right] &= \frac{1}{h} \left[\frac{x^2}{x^2(x + h)^2} - \frac{(x + h)^2}{x^2(x + h)^2} \right] = \frac{1}{h} \left[\frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x + h)^2} \right] \\ &= \frac{1}{h} \left[\frac{x^2 - x^2 - 2xh - h^2}{x^2(x + h)^2} \right] = \frac{1}{h} \left[\frac{-2xh - h^2}{x^2(x + h)^2} \right] = \frac{1}{h} \cdot \frac{h(-2x - h)}{x^2(x + h)^2} = \frac{-2x - h}{x^2(x + h)^2} \end{aligned}$$

8. (R8) Factor the expression $x^{1/2}(x^2 + x) + x^{3/2} - 24x^{1/2}$ (where $x \geq 0$).

Notice that $x^{1/2}$ is a factor of each term, since $x^{3/2} = x^{1+1/2} = x^{1/2} \cdot x$. The answer is (d).

$$x^{1/2}(x^2 + x) + x^{3/2} - 24x^{1/2} = x^{1/2}(x^2 + x + x - 24) = x^{1/2}(x^2 + 2x - 24) = x^{1/2}(x + 6)(x - 4)$$

9. (R8) Multiply and simplify: $(2\sqrt{x} - 3)(2\sqrt{x} + 5)$

$$\begin{aligned} (2\sqrt{x} - 3)(2\sqrt{x} + 5) &= 2\sqrt{x}(2\sqrt{x} + 5) - 3(2\sqrt{x} + 5) = (2\sqrt{x})^2 + 5(2\sqrt{x}) - 3(2\sqrt{x}) - 15 \\ &= 4x + 10\sqrt{x} - 6\sqrt{x} - 15 = 4x + 4\sqrt{x} - 15 \quad \text{Answer: (a)} \end{aligned}$$

10. (R8 Example 7) Simplify $(x^{2/3}y)(x^{-2}y)^{1/2}$: Answer: (d) (See page 74 of the text.)

11. (R8 51) After rationalizing the denominator of $\frac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$, the numerator is ???

$$\begin{aligned} \frac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} &= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} - \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})(\sqrt{x+h} - \sqrt{x})} = \frac{(\sqrt{x+h})^2 - 2\sqrt{x}\sqrt{x+h} + (\sqrt{x})^2}{(\sqrt{x+h})^2 - (\sqrt{x})^2} \\ &= \frac{x+h - 2\sqrt{x}\sqrt{x+h} + x}{x+h-x} = \frac{2x+h - 2\sqrt{x}\sqrt{x+h} + x}{h} \quad \text{Answer: (b) } 2x+h - 2\sqrt{x(x+h)} \end{aligned}$$

12. (1.1 49) Solve the equation $\frac{x}{x-2} + 3 = \frac{2}{x-2}$. As long as $x \neq 2$, we can multiply both sides by $x-2$.

$$(x-2) \left(\frac{x}{x-2} + 3 \right) = (x-2) \cdot \frac{2}{x-2} \quad x+3(x-2) = 2 \quad 4x = 8 \quad x = 2$$

This does *not* lead to a solution, since we had to exclude $x = 2$ before we could use the term $x-2$ to simplify the equation. Another way to see that the correct answer is (d) NO SOLUTION is to try to substitute $x = 2$ into the equation to check. You will have division by zero, which doesn't make sense.

13. (1.2) The equation $1 - \frac{1}{x} - \frac{12}{x^2} = 0$ has (d) exactly TWO real solutions, whose product is -12

The answer is phrased this way so you can't just use trial and error, by substituting in the various choices.

$$x^2 \left(1 - \frac{1}{x} - \frac{12}{x^2} \right) = x^2 \cdot 0 \quad x^2 - x - 12 = 0 \quad (x-4)(x+3) = 0 \quad x = 4 \text{ or } x = -3$$

14. (1.2 51) Use the quadratic formula to solve the equation $\frac{2}{3}x^2 - \frac{5}{3}x + 1 = 0$. The solutions are ???

Multiply through by 3 to get $2x^2 - 5x + 3 = 0$. The quadratic formula applies to an equation of the form $ax^2 + bx + c = 0$, so in this case $a = 2$, $b = -5$, and $c = 3$. Then you need to substitute into the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \text{ You get } x = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4} \text{ so } x = \frac{6}{4} = \frac{3}{2} \text{ or } x = \frac{4}{4} = 1.$$

Answer: (e) None of these

15. (1.2) Find the value of k so that $x^2 - \frac{3}{2}x + k$ is a perfect square.

$$\text{Divide } -\frac{3}{2} \text{ by 2 to get } -\frac{3}{4} \text{ and then square to get the answer: (c) } k = \frac{9}{16}.$$

16. (1.4 25) Find the real solutions of the equation $3 + \sqrt{3x+1} = x$. Answer: (e) None of these

You need to find a way to simplify the square root. The easiest way to do this is to subtract 3 from both sides of the equation, and then square both sides. $(\sqrt{3x+1})^2 = (x-3)^2 \quad 3x+1 = x^2 - 6x + 9$

$$0 = x^2 - 9x + 8 \quad (x-1)(x-8) = 0 \quad x = 1 \text{ or } x = 8$$

The next step is to substitute these possible answers into the *original* equation. Because we squared both sides of the equation, we may have caused problems with signs, and so it is possible that the answers we got after squaring won't work in the original form of the equation.

Test $x = 1$: On the left hand side of the original equation we get $3 + \sqrt{3+1} = 3 + \sqrt{4} = 3 + 2 = 5$. On the right hand side of the equation we get 1, so this does *not* give us a solution.

Test $x = 8$: On the left hand side of the equation we get $3 + \sqrt{24+1} = 3 + \sqrt{25} = 3 + 5 = 8$. On the right hand side of the equation we also get 8, so $x = 8$ is the only solution.