This sample has 42 questions: the actual exam will have 40!

**Directions:** On the answer sheet, fill in the following in the appropriate spaces and darken the corresponding ovals:

1. Last name, first and middle initials.

2. Student Z Number. (LEFT-justify the 6 digits in the ID field leaving the last 3 spaces blank.)

3. Section:
   - A1=11, A3=13
   - B1=21, B2=22
   - C1=31, C2=32
   - D1=41, D2=42
   - E1=51, D2=52
   - G1=71, G2=72

4. Your signature on the back.

**Problems:** Check that your exam contains exactly 40 problems. Each problem is worth 5 points. Circle the answer on the exam and darken the corresponding oval on the answer sheet.

1. Factor the expression \( x^{1/2}(x^2 + x) - 3x^{3/2} - 3x^{1/2} \) (where \( x \geq 0 \)).
   - (a) \( x^{1/2}(x + 1)(x - 3) \)
   - (b) \( x^{3/2}(x + 1)(x - 3) \)
   - (c) \( x^{3/2}(x + 2)(x - 6) \)
   - (d) \( x^{1/2}(x + 6)(x - 4) \)
   - (e) \( (x + 6)(x - 4) \)

2. What is the domain of the function \( f \) defined by \( f(x) = \frac{x + 2}{\sqrt{5 + 3x}} \)?
   - (a) \( (-\infty, 0] \)
   - (b) \( (-\infty, 0) \)
   - (c) \( (-2, \infty) \)
   - (d) \( (-5/3, \infty) \)
   - (e) \( (-\infty, -5/3) \)
3. Find the domain of \( \log(4x^3 - x^5) \).

(a) \( \{ x : -2 < x < 2 \} \)  
(b) \( \{ x : -2 < x < 0 \text{ or } x > 2 \} \)  
(c) \( \{ x : x < 2 \} \)  
(d) \( \{ x : x < -2 \text{ or } 0 < x < 2 \} \)  
(e) \( \{ x : x < -2 \text{ or } x > 2 \} \)

4. Simplify \( \frac{2 \sqrt{5} + 1}{2 \sqrt{5} - 1} \)

(a) 1  
(b) 4  
(c) 16  
(d) \( 2^{\sqrt{5}} \)  
(e) 32

5. If \( f(t) = \frac{1}{t} + 1 \), then \( f^{-1}(t) = \)

(a) \( \frac{-1}{t} + 1 \)  
(b) \( \frac{1}{t} - 1 \)  
(c) \( \frac{1}{t + 1} \)  
(d) \( \frac{1}{t - 1} \)  
(e) \( \frac{t}{t + 1} \)

6. If \( f(x) = \frac{x}{x^2 + 3} \), then \( f(3 + h) = \)

(a) \( 3 + \frac{h}{h^2 + 3} \)  
(b) \( \frac{3}{3^2 + 3} + \frac{h}{h^2 + 3} \)  
(c) 3  
(d) \( \frac{h + 3}{h^2 + 3} \)  
(e) \( \frac{h + 3}{(h + 3)^2 + 3} \)
7. What is the average rate of change of \( s(t) = 7t^2 \) on \([2, 5]\)?

(a) 7  (d) 147

(b) 49  (e) None of the above

(c) 63

8. Find the solution(s) of the equation \( e^{3x+5} = 2 \).

(a) \(-1\)  (d) \(\frac{\ln 2 - 5}{3}\)

(b) 1  (e) \(\ln 0 + \ln 2 - 5\)

(c) \(\frac{-4 \ln 3}{3}\)

9. Solve \( 5^{-4x} = 2^{x+3} \)

(a) \(x = 1\)  (d) \(x = \frac{(-7 \ln 2)}{(3 \ln 2 - 4 \ln 5)}\)

(b) \(x = \frac{(-7 \ln 2)}{(3 \ln 2 + 4 \ln 5)}\)  (e) \(x = \frac{7 \ln 2}{(3 \ln 2 + 4 \ln 5)}\)

(c) \(x = \frac{-2}{13}\)

10. Find the composition \( f \circ g(x) \) when

\[
f(x) = \frac{x + 6}{3x - 6} \quad \text{and} \quad g(x) = \frac{x + 1}{x - 2}.
\]

(a) \(\frac{4x}{-5x + 18}\)  (d) \(\frac{x + 6}{3x + 3}\)

(b) \(\frac{7x - 11}{-3x + 15}\)  (e) none of these

(c) \(\frac{3x + 3}{x + 6}\)
11. To obtain the graph of \( f(x) = (5 - x)^3 \) we can

(a) shift the graph of the basic function \( g(x) = x^3 \) right 5 units then reflect around the \( x \)-axis.
(b) shift the graph of the basic function \( g(x) = x^3 \) up 5 units then reflect around the \( y \)-axis.
(c) shift the graph of the basic function \( g(x) = x^3 \) left 5 units then reflect around the \( y \)-axis.
(d) shift the graph of the basic function \( g(x) = x^3 \) left 5 units then reflect around the \( x \)-axis.
(e) None of the these

12. A wire of length \( x \) is bent into the shape of a square. Express the area \( A \) of the square as a function of \( x \).

(a) \( A(x) = x^2 \)  
(b) \( A(x) = 4x^2 \)  
(c) \( A(x) = \frac{x^2}{4} \)  
(d) \( A(x) = 2x^2 \)  
(e) none of these

13. Solve \( 3(2^{2x}) - 5(2^x) + 2 = 0 \)

(a) \( x = 0 \) is the only solution  
(b) \( x = 1 \) is the only solution  
(c) \( x = 2 \) is the only solution  
(d) \( x = 0 \) or \( x = 1 - \log_2(3) \)  
(e) \( x = 1 \) or \( x = 1 - \log_2(3) \)

14. The population of a midwestern city follows the exponential law. If the population decreased from 1,000,000 to 800,000 from 1995 to 1998, what will the population be in 2001?

(a) 640,000  
(b) 600,000  
(c) 480,000  
(d) 400,000  
(e) None of the above
15. Simplify \((\tan x) \left( \frac{1}{\sin x} - \sin x \right)\)

(a) \(\cos x\)  \hspace{1cm} (d) \(\sin x\)
(b) \(1\)  \hspace{1cm} (e) \(\tan x\)
(c) \(0\)

16. Which of the following are identities?

(I) \(\cos^2 t - \sin^2 t = 1 + 2\sin^2 t\)
(II) \(\tan \beta \sin \beta = \sec \beta + \cos \beta\)
(III) \(\csc \theta \cos \theta \tan \theta = 1\)

(a) I, II, III  \hspace{1cm} (d) III only
(b) I only  \hspace{1cm} (e) Some other selection
(c) II only

17. Suppose that \(\triangle ABC\) is a right triangle with \(\angle C = \frac{\pi}{2}\). If \(AC = 6\) and \(BC = 8\) then:

(a) \(\sin A = \frac{4}{5} \& \cos A = \frac{3}{5} \& \tan A = \frac{3}{4}\)  \hspace{1cm} (d) \(\sin A = \frac{3}{5} \& \cos A = \frac{4}{5} \& \tan A = \frac{3}{4}\)
(b) \(\sin A = \frac{4}{5} \& \cos A = \frac{3}{5} \& \tan A = \frac{4}{3}\)  \hspace{1cm} (e) None of the above are true
(c) \(\sin A = \frac{3}{4} \& \cos A = \frac{4}{5} \& \tan A = \frac{4}{5}\)
18. $450^\circ =$

(a) $\frac{\pi}{3}$  
(b) $\frac{5\pi}{2}$  
(c) $\frac{2\pi}{5}$  
(d) $5\pi$  
(e) Some other value.

19. Two angles of a triangle are $\frac{\pi}{3}$ and $\frac{2\pi}{5}$. What is the third angle?

(a) $\frac{4\pi}{15}$  
(b) $\frac{\pi}{2}$  
(c) $\frac{2\pi}{5}$  
(d) $\frac{7\pi}{5}$  
(e) Some other value.

20. Evaluate $\sin\left(\frac{183\pi}{4}\right)$

(a) $\frac{\sqrt{2}}{2}$  
(b) $-\frac{\sqrt{2}}{2}$  
(c) $\frac{1}{2}$  
(d) $-\frac{1}{2}$  
(e) None of the above
21. In the following graph of $y = \sin(x)$ the point $P$ has coordinates:

(a) $\left(\frac{\pi}{2}, -2\right)$  
(b) $\left(\frac{\pi}{2}, -1\right)$  
(c) $\left(\frac{5\pi}{2}, -1\right)$  
(d) $\left(\frac{7\pi}{2}, -1\right)$  
(e) $(7\pi, -1)$

22. Simplify $\frac{\sin^2 \beta}{1 - \cos \beta} + \frac{\sin^2 \beta}{1 + \cos \beta}$

(a) $2$  
(b) $2 \cos^2 \beta$  
(c) $2 \cos^2 \beta$  
(d) $2 \sin \beta \cos \beta$  
(e) $2 \tan^2 \beta$
23. When an analog clock reads 5:00, what is the smaller angle, in radians, between the hour hand and the minute hand?

(a) \( \frac{\pi}{2} \)  \quad (d) \( \frac{\pi}{12} \)
(b) \( \frac{5\pi}{6} \)  \quad (e) None of the above
(c) \( \frac{7\pi}{6} \)

24. A wheel of radius 3 feet is rotating at 100 rpm (revolutions per minute). What is the linear speed in feet per minute of a point on the circumference of the wheel?

(a) 200\pi \text{ ft/min}  \quad (c) 900 \text{ ft/min}  \quad (e) None of the above
(b) 600 \text{ ft/min}  \quad (d) 200 \text{ ft/min}

25. What is the area of the triangle with sides \( a = 4 \), \( b = 10 \) and included angle \( \theta = \frac{\pi}{6} \)?

(a) 5  \quad (d) 40
(b) 10  \quad (e) None of the above.
(c) 20
26. Which is the graph of $y = \cos x$?

(a)  

(b)  

(c)  

(d)  

(e)  

27. What is the period of $f(x) = -3 \sin(13\pi x - 2)$?

(a) 3  

(b) $\frac{2}{13}$  

(c) $\frac{2\pi}{13}$  

(d) 2  

(e) None of the above
For the next two problems:

\[
\sin \alpha = \frac{1}{3} \text{ with } \frac{\pi}{2} < \alpha < \pi \quad \text{and} \quad \sin \beta = -\frac{2}{7} \text{ with } \pi < \beta < \frac{3\pi}{2}
\]

28. Find the exact value of \( \cos(\alpha - \beta) \):

(a) \( \frac{6\sqrt{10} - 2}{21} \)

(b) \( \frac{-3\sqrt{5} - 4\sqrt{2}}{21} \)

(c) \( \frac{6\sqrt{10} + 2}{21} \)

(d) \( \frac{-3\sqrt{5} + 4\sqrt{2}}{21} \)

(e) None of the above

29. Find the exact value of \( \sin(\alpha - \beta) \):

(a) \( \frac{6\sqrt{10} - 2}{21} \)

(b) \( \frac{-3\sqrt{5} - 4\sqrt{2}}{21} \)

(c) \( \frac{6\sqrt{10} + 2}{21} \)

(d) \( \frac{-3\sqrt{5} + 4\sqrt{2}}{21} \)

(e) None of the above

30. Find the exact value of \( \cos(-15^\circ) \):

(a) \( \frac{\sqrt{2} + \sqrt{3}}{2\sqrt{2}} \)

(b) \( \frac{-\sqrt{2} + \sqrt{3}}{2\sqrt{2}} \)

(c) \( \frac{\sqrt{2} - \sqrt{3}}{2\sqrt{2}} \)

(d) \( \frac{\sqrt{2} + \sqrt{2}}{2\sqrt{3}} \)

(e) None of the above

31. Solve \( \sin(x) \cos(x) = \frac{\sqrt{2}}{4} \):

(a) \( x = \frac{\pi}{4} \)

(b) \( x = \frac{\pi}{4} + 2k\pi \) or \( \frac{3\pi}{4} + 2k\pi \)

(c) \( x = \frac{\pi}{8} + k\pi \) or \( \frac{3\pi}{8} + k\pi \)

(d) \( x = \pm \frac{\pi}{8} + 2k\pi \)

(e) None of the above
32. Find all angles $\theta$ in the range $0^\circ \leq \theta \leq 360^\circ$ for which $\sin(\theta) = \sqrt{3}\cos(\theta)$

(a) $\theta = 60^\circ$  
(c) $\theta = 30^\circ$  
(e) None of the above  

(b) $\theta = 60^\circ$ or $240^\circ$  
(d) $\theta = 30^\circ$ or $210^\circ$

33. Simplify $\cos^{-1}(\cos(\frac{5\pi}{4}))$

(a) $\frac{5\pi}{4}$  
(c) $-\frac{\pi}{4}$  
(e) None of the above  

(b) $\frac{\pi}{4}$  
(d) $\frac{3\pi}{4}$

34. Solve $\sin^2(\theta) + 3\sin(\theta) = -2$.

(a) Either $\theta = \sin^{-1}(-1)$ or $\theta = \sin^{-1}(-2)$  
(d) $\theta = -\frac{(2k + 1)\pi}{2}$

(b) $\theta = \sin^{-1}(-1)$  
(e) None of the above.

(c) $\theta = -\frac{\pi}{2} + 2k\pi$

35. Simplify $\cos(\cos^{-1}\left(\frac{5\pi}{40}\right))$

(a) $\frac{5\pi}{40}$  
(c) $\frac{35\pi}{40}$  
(e) None of the above.

(b) $-\frac{5\pi}{40}$  
(d) $\frac{25\pi}{40}$
36. I want to construct a triangle with sides of length $a, b, c$ opposite angles $\alpha, \beta, \gamma$ respectively.
If I want $a = 2, b = 5$ and $c = 12$:

(a) $\cos(\alpha) = \frac{2^2 + 5^2 + 12^2}{2 \cdot 5 \cdot 12}$
(d) There is such a triangle but $\cos(\alpha)$ is not as above.

(b) $\cos(\alpha) = \frac{2^2 + 5^2 - 12^2}{2 \cdot 5 \cdot 12}$
(e) No such triangle is possible.

(c) $\cos(\alpha) = \frac{5^2 + 12^2 - 2^2}{2 \cdot 5 \cdot 12}$

37. I want to construct a triangle with sides of length $a, b, c$ opposite angles $\alpha, \beta, \gamma$ respectively.
If I want $\sin(\alpha) = \frac{1}{10}, \sin(\beta) = \frac{1}{8}$ and $b = 5$:

(a) $a = 4$ (d) There is such a triangle but $a$ is not as above.

(b) $a = \frac{1}{4}$ (e) No such triangle is possible.

(c) $a = \frac{1}{20}$

38. Convert polar coordinates of $(6, \frac{\pi}{6})$ to rectangular coordinates:

(a) $(3, 3\sqrt{2})$ (d) $(3, 3\sqrt{2})$

(b) $(3\sqrt{2}, 3\sqrt{3})$ (e) None of the above.

(c) $(6, 3\sqrt{3})$

39. Convert rectangular coordinates of $(6, -6)$ to polar coordinates:

(a) The only possible polar coordinates are $(6, -\frac{\pi}{4})$

(b) The only possible polar coordinates are $(6\sqrt{2}, -\frac{\pi}{4})$

(c) Possible polar coordinates are $(6\sqrt{2}, -\frac{\pi}{4})$ and $(-6\sqrt{2}, \frac{3\pi}{4})$

(d) Possible polar coordinates are $(6\sqrt{2}, -\frac{\pi}{4})$ and $(-6\sqrt{2}, -\frac{3\pi}{4})$

(e) None of the above.
40. The graph of $-7x^2 + 5y^2 + 23x - 46y + 7 = 0$ is:
   (a) a circle  (d) an ellipse (Not circle)
   (b) a line  (e) a hyperbola
   (c) a parabola

41. Find the vertex of the parabola $7y = x^2 - 12x + 57$.
   (a) (3, 6)  (d) $(-3, -6)$
   (b) (6, 3)  (e) None of the above.
   (c) $(-3, 6)$

42. Which of the following is the graph of $r = -4 \sin \theta$?

(a)  
(b)  
(c)  
(d)  
(e)  
