**MATH 155**

**FINAL - FORM A**

**Solutions**

Name: ____________  Section: ____________  Zid: ____________

**Directions:** Complete the information above then, on the answer sheet, fill in the following in the appropriate spaces and darken the corresponding ovals:

1. Last name, first and middle initials.

2. Student Z Number. (LEFT-justify the 6 digits in the ID field leaving the last 3 spaces blank.)

3. Section:

   A1=11  B1=21  C1=31  D1=41
   A2=12  B2=22  C2=32  D2=42
   A3=13  B3=23  C3=33  D3=43

4. Your signature on the back.

5. No Scratch paper outside of the Exam is permitted.

6. Only a basic non-text capable, non-graphing calculator is permitted.

7. **Graphing calculators, cell phones and pdas shall be stowed out of sight.**
   IF VISIBLE YOU WILL BE DEEMED TO BE CHEATING AND WILL RECEIVE A ZERO SCORE FOR THE EXAM!!

8. Check that your exam contains exactly 40 problems. Each problem is worth 5 points.

**Problem 1A**

Factor the expression \(x^{2/3}(x^2 + 8x) - 5x^{5/3} - 28x^{3/3}\).

\[
= x^{2/3} + 8x^{5/3} - 5x^{5/3} - 28x^{3/3} = \quad \frac{8x^{2/3}}{x^{5/3}} + \frac{5x^{2/3}}{x^{5/3}} - 28x^{3/3}
\]

(a) \(x^{5/3}(x - 4)(x + 7)\)

(b) \(x^{5/3}(x + 4)(x - 7)\)

(c) \(x^{2/3}(x - 4)(x + 7)\)


**Problem 2A**

Evaluate \(\log_b(1)\), \(b > 0, b \neq 1\)

(a) 0  
(b) 1  
(c) \(b\)

(d) e  
(e) None of the above.

\[\log_b 1 = 0 \text{ because } b^0 = 1 \text{ for any real } b.\]
[3A] Simplify \( \frac{5\sqrt{3} + 1}{5\sqrt{3} - 1} = \frac{5\sqrt{3} + 1}{5\sqrt{3} - 1} \times \frac{5\sqrt{3} + 1}{5\sqrt{3} + 1} = \frac{5^2 - 1}{5^2 - 1} = \frac{25}{25} = 1 \)

(a) 1
(b) 25
(c) \( \frac{1}{25} \)
(d) \( 5^2\sqrt{3} \)
(e) None of the above.

[4A] If \( f(t) = \frac{3}{t+1} \), then \( f^{-1}(t) = \)

(4) \( \frac{3}{y+1} \)
(4) \( t \)
(4) \( y(t+1) = 3 \)
(4) \( ty + t = 3 \)
(4) \( y = \frac{3 - t}{t} = \frac{3}{t} - 1 \)
(5) \( f^{-1}(t) = \frac{3}{t} - 1 \)

(a) \( \frac{3}{t+1} \)
(b) \( \frac{-3}{t+1} \)
(c) \( \frac{3}{t} - 1 \)
(d) \( \frac{3}{t+1} \)
(e) None of the above.

[5A] Which is the largest?

(a) \( \ln(e^3) = 3 \) \( (e^3 = 3) \)
(b) \( \log_3(17) > 4 \) \( (2^4 = 16) \)
(c) \( \log_5(990) < 3 \) \( (10^3 = 1000) \)
(d) \( \log_5(63) < 2 \)
(e) 1

\( 8^2 = 64 \)

[6A] If \( f(x) = x^2 - 5x \) then \( f(h+2) = (h+2)^2 - 5(h+2) \)

(a) \( h^2 - 5h - 6 \)
(b) \( h^2 + 5h + 14 \)
(c) \( h^2 + h - 6 \)
(d) \( h^2 - h - 6 \)
(e) None of the above.

[7A] What is the average rate of change of \( s(t) = 2t^2 - 3 \) on \([3, 5] \)?

(a) 16
(b) 20
(c) 32
(d) 40
(e) None of the above

\( = \frac{s(5) - s(3)}{5 - 3} \)

\( = \frac{2(5)^2 - 3 - [2(3)^2 - 3]}{5 - 3} \)

\( = \frac{(25 - 3) - (15)}{2} = \frac{47 - 15}{2} = \frac{32}{2} = 16 \)
[8A] Find the solution(s) of the equation \( e^{5x-7} = 6 \).

(a) \( \frac{\ln 6 + 5}{7} \)

(b) \( \frac{\ln 7 + 6}{5} \)

(c) \( \frac{\ln 7 + 5}{6} \)

(d) \( \frac{\ln 6 + 7}{5} \)

(e) None of the above

\[ 5x - 7 = \ln 6 \]
\[ 5x = \ln 6 + 7 \]
\[ x = \frac{\ln (6 + 7)}{5} \]

[9A] If \( f(x) = 2x - 5 \) and \( g(x) = 3x + 4 \), find the product \( (gf)(x) \) and the composition \( g \circ f(x) \).

(a) \( (gf)(x) = 6x - 11 \) and \( g \circ f(x) = 6x^2 - 7x - 20 \)

(b) \( (gf)(x) = 6x^2 - 7x - 20 \) and \( g \circ f(x) = 6x - 11 \)

(c) \( (gf)(x) = 6x + 3 \) and \( g \circ f(x) = 6x^2 - 7x - 20 \)

(d) \( (gf)(x) = 6x^2 - 7x - 20 \) and \( g \circ f(x) = 6x + 3 \)

(e) None of these

\[ (g \circ f)(x) = (2x - 5)(3x + 4) \]
\[ = 6x^2 + 8x - 15x - 20 \]
\[ = 6x^2 - 7x - 20 \]

\[ (g \circ f)(x) = 3(2x - 5) + 4 \]
\[ = 6x - 15 + 4 \]
\[ = 6x - 11 \]

[10A] Let \( F(x) \) be a general exponential function \( F(x) = b^x \), \( b > 0, b \neq 1 \) and let \( G(x) \) be a general logarithm function \( G(x) = \log_b(x) \), \( b > 0, b \neq 1 \).

Which of the following is always true?

- Both functions are increasing
- Both functions are decreasing
- The y-axis is an asymptote for \( F(x) \) and the x-axis is an asymptote for \( G(x) \)
- The graph of \( F(x) \) crosses the x-axis at \( x = 1 \) and that of \( G(x) \) crosses the y-axis at \( y = 1 \) (vice-versa)
- The graph of \( F(x) \) crosses the y-axis at \( y = 1 \) and that of \( G(x) \) crosses the x-axis at \( x = 1 \)

[11A] Let \( f(x) = -g(x + 3) \) for some function \( g(x) \).

To obtain the graph of \( f(x) \) we can:

(a) shift the graph of \( g(x) \) right 3 units then reflect around the y-axis.

(b) shift the graph of \( g(x) \) left 3 units then reflect around the y-axis.

(c) shift the graph of \( g(x) \) right 3 units then reflect around the x-axis.

(d) shift the graph of \( g(x) \) left 3 units then reflect around the x-axis.

(e) None of the above
[12A] A wire of length $2x$ is bent into the shape of a square. Express the area $A$ of the square as a function of $x$.

(a) $A(x) = x^2$
(b) $A(x) = 4x^2$
(c) $A(x) = \frac{x^2}{2}$
(d) $A(x) = 2x^2$
(e) None of these

[13A] Solve $2(3^{2x}) + 3(3^x) - 5 = 0$

(a) $x = 0$ is the only solution
(b) $x = 1$ is the only solution
(c) $x = 0$ or $x = 1$
(d) No solution
(e) None of the above

[14A] What is the domain of the function $f(x) = \frac{x + 5}{1 + \sqrt{9 - x^2}}$?

(a) $(-\infty, 5)$
(b) $(-3, \infty)$
(c) $[-3, 3]$ (e) None of the above

[15A] The population of a midwestern city follows the exponential law. If the population increased from 10,000 to 30,000 from 2004 to 2006, what will the population be in 2008?

(a) 60,000
(b) 90,000
(c) 120,000
(d) 270,000
(e) None of the above

[16A] Which function has amplitude 7, period $\frac{2\pi}{5}$ and phase shift $\frac{19\pi}{5}$?

(a) $f(x) = -7 \sin(5x - 19\pi)$
(b) $f(x) = -7 \sin(19x + 5\pi)$
(c) $f(x) = -7 \sin(5x + 19\pi)$
(d) $f(x) = -5 \sin(19x - 7\pi)$
(e) $f(x) = -5 \sin(7x - 19\pi)$
[17A] Simplify and evaluate \( \sin \left( \frac{13\pi}{24} \right) \cos \left( \frac{5\pi}{24} \right) - \cos \left( \frac{13\pi}{24} \right) \sin \left( \frac{5\pi}{24} \right) \) as \( \sin \left( \frac{13\pi}{24} - \frac{5\pi}{24} \right) \) or \( \sin \left( \frac{8\pi}{24} \right) \) or \( \sin \left( \frac{\pi}{3} \right) \)

(a) \( \frac{\sqrt{3}}{2} \)  
(b) \( -\frac{\sqrt{3}}{2} \)  
(c) \( \frac{\sqrt{2}}{2} \)  
(d) \( -\frac{\sqrt{2}}{2} \)  
(e) \( \frac{1}{2} \)

[18A] Solve \( 2\cos^2(\theta) - 2\sin^2(\theta) = \sqrt{3} \) as \( \cos^2(\theta) - \sin^2(\theta) = \frac{\sqrt{3}}{2} \)

(a) \( \theta = \pm \frac{\pi}{6} + k\pi \)  
(b) \( \theta = \pm \frac{\pi}{3} + k\pi \)  
(c) \( \theta = \frac{\pi}{3} \)  
(d) \( \theta = \pm \frac{\pi}{12} + k\pi \)  
(e) None of the above.

\[ 2\theta = \frac{\pi}{6} \text{ or } -\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z} \]
\[ \theta = \pm \frac{\pi}{12} + k\pi, \quad k \in \mathbb{Z} \]

[19A] In the following graph of \( y = \tan(x) \) the point \( Q \) has coordinates:

(a) \( \left( \frac{\pi}{2}, 0 \right) \)  
(b) \( (\pi, 0) \)  
(c) \( \left( \frac{3\pi}{2}, 0 \right) \)  
(d) \( \left( \frac{7\pi}{2}, 0 \right) \)  
(e) \( (2\pi, 0) \)
20A] If \( \sin \alpha = -\frac{2}{3} \) with \( \frac{\pi}{3} < \alpha < \frac{3\pi}{2} \) and \( \sin \beta = \frac{1}{5} \) with \( \frac{\pi}{2} < \beta < \pi \):

Find the exact value of \( \cos(\alpha + \beta) \):

\[
\frac{4\sqrt{6} + \sqrt{5}}{15} \quad (c) \quad \frac{2\sqrt{30} + 2}{15} \quad (e) \quad \text{None of the above}
\]

\[
\frac{4\sqrt{6} - \sqrt{5}}{15} \quad (d) \quad \frac{2\sqrt{30} - 2}{15}
\]

21A] Simplify \( \cos^{-1}\left(\cos \left[\frac{-\pi}{5}\right]\right) \)

\[
\cos^{-1}(\cos \frac{-\pi}{5}) = \frac{-\pi}{5} \quad \text{range of } \cos^{-1}(x) \text{ is } 0 \text{ to } \pi.
\]

\[
(a) \ -\frac{\pi}{5} \quad (c) \ -\frac{4\pi}{5} \quad (e) \ \text{None of the above}
\]

\[
(b) \ \frac{\pi}{5} \quad (d) \ \frac{4\pi}{5}
\]

22A] Simplify \( \cos \left(\cos^{-1} \left[\frac{-\pi}{5}\right]\right) \)

\(-\frac{\pi}{5} \) is between \(-1\) and \(1\), so it’s within the domain of \( \cos^{-1}(x) \), so \( \cos(\cos^{-1}(-\frac{\pi}{5})) \)

\[
(a) \ -\frac{\pi}{5} \quad (c) \ -\frac{4\pi}{5} \quad (e) \ \text{None of the above}
\]

\[
(b) \ \frac{\pi}{5} \quad (d) \ \frac{4\pi}{5}
\]

23A] I want to construct a triangle with sides of length \(a, b, c\) opposite angles \(\alpha, \beta, \gamma\) respectively.

If I want \(a = 5, b = 4\) and \(c = 7\):

\[
\frac{s^2}{a} = \frac{4^2 + 7^2 - 2 \cdot 4 \cdot 7 \cos \alpha}{2 \cdot 4 \cdot 5}
\]

\[
\cos(\alpha) = \frac{4^2 + 7^2 - 5^2}{2 \cdot 4 \cdot 7}
\]

\[
\cos(\alpha) = \frac{4^2 + 5^2 - 7^2}{2 \cdot 4 \cdot 5}
\]

\[
(a) \ \cos(\alpha) = \frac{4^2 + 5^2 - 7^2}{2 \cdot 4 \cdot 5} \quad (d) \ \text{There is such a triangle but } \cos(\alpha) \text{ is not as above.}
\]

\[
(b) \ \cos(\alpha) = \frac{4^2 + 7^2 - 5^2}{2 \cdot 4 \cdot 7}
\]

\[
(c) \ \cos(\alpha) = \frac{4^2 + 7^2 - 5^2}{2 \cdot 4 \cdot 7}
\]

\[
(a) \ \cos(\alpha) = \frac{4^2 + 7^2 - 5^2}{2 \cdot 4 \cdot 7} \quad (e) \ \text{No such triangle is possible.}
\]

24A] I want to construct a triangle with sides of length \(a, b, c\) opposite angles \(\alpha, \beta, \gamma\) respectively.

If I want \(\sin(\alpha) = \frac{1}{6}, \ \sin(\beta) = \frac{1}{10}\) and \(b = 12\) then:

\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}
\]

\[
(a) \ a = 4 \quad (c) \ a = 9 \quad (e) \ \text{None of the above.}
\]

\[
(b) \ a = 5 \quad (d) \ a = 20
\]

\[
\frac{(\frac{1}{6})}{a} = \frac{(\frac{1}{10})}{12}
\]

\[
2 = \frac{a}{10}
\]

\[
a = 20
\]
[25A] I want to construct a triangle with sides of length $a, b, c$ opposite angles $\alpha, \beta, \gamma$ respectively such that $b = 5$, $a = 10$ and $\beta = \sin^{-1}\left(\frac{1}{4}\right)$. Noting that $\frac{\pi}{6} + \beta \approx 0.25\pi$ and $\frac{5\pi}{6} + \beta \approx 0.91\pi$, necessarily:

(a) $\alpha = \frac{\pi}{6}$
(b) $\alpha = \frac{5\pi}{6}$
(c) Either $\alpha = \frac{\pi}{6}$ or $\alpha = \frac{5\pi}{6}$

(d) There is such a triangle but $\alpha$ is not as above.
(e) No such triangle is possible.

[26A] Solve $2\cos^2(\theta) + \cos(\theta) = 3$.

(a) Either $\theta = \cos^{-1}(1)$ or $\theta = \cos^{-1}\left(-\frac{3}{2}\right)$
(b) Either $\theta = \cos^{-1}(1)$ or $\theta = \cos^{-1}\left(-\frac{5}{2}\right)$
(c) $\theta = \cos^{-1}(1)$

[27A] Convert polar coordinates of $(4, -\frac{\pi}{3})$ to rectangular coordinates:

(a) $(2, 2\sqrt{3})$
(b) $(2, -2\sqrt{3})$
(c) $(2\sqrt{3}, 2)$

(d) $(-2\sqrt{3}, 2)$
(e) None of the above.

[28A] Convert rectangular coordinates of $(-4, 4)$ to polar coordinates:

(a) The only possible polar coordinates are $(4\sqrt{2}, \frac{5\pi}{4})$
(b) The only possible polar coordinates are $(4\sqrt{2}, -\frac{5\pi}{4})$
(c) Possible polar coordinates are $(4\sqrt{2}, \frac{5\pi}{4})$ and $(-4\sqrt{2}, \frac{\pi}{4})$
(d) Possible polar coordinates are $(4\sqrt{2}, \frac{5\pi}{4})$ and $(4\sqrt{2}, -\frac{\pi}{4})$
(e) None of the above.
[29A] Find all angles $\theta$ in the range $0^\circ \leq \theta \leq 360^\circ$ for which $\sqrt{3} \sin(\theta) = \cos(\theta)$.

(a) $\theta = 30^\circ$  \hspace{1cm} (c) $\theta = 60^\circ$ or $240^\circ$  \hspace{1cm} (e) None of the above

(b) $\theta = 60^\circ$  \hspace{1cm} (d) $\theta = 30^\circ$ or $210^\circ$

[30A] Suppose that $\triangle ABC$ is a right triangle with $\angle C = \frac{\pi}{2}$. If $AC = 10$ and $BC = 24$ then:

(a) $\cos B = \frac{12}{13}$ & $\sin B = \frac{5}{13}$ & $\tan A = \frac{5}{12}$

(b) $\cos B = \frac{12}{13}$ & $\sin B = \frac{5}{13}$ & $\tan A = \frac{12}{5}$

(c) $\cos B = \frac{5}{13}$ & $\sin B = \frac{12}{13}$ & $\tan A = \frac{12}{5}$

(d) $\cos B = \frac{5}{13}$ & $\sin B = \frac{12}{13}$ & $\tan A = \frac{5}{12}$

(e) None of the above are true.

[31A] Two angles of a triangle are $\frac{3\pi}{8}$ and $\frac{1\pi}{8}$. What is the third angle?

(a) $\frac{29\pi}{40}$

(b) $\frac{23\pi}{40}$

(c) $\frac{17\pi}{40}$

(d) $\frac{11\pi}{40}$

(e) None of the above.

[32A] Find the area of the sector of radius 4 in. and central angle $20^\circ$.

$[A = \frac{1}{2} \theta r^2$ when the angle is in radians.$]$

(a) $\frac{4\pi}{9}$ in.$^2$.  \hspace{1cm} (d) 80 in.$^2$.  \hspace{1cm} $r = 4$

(b) $\frac{8\pi}{9}$ in.$^2$.  \hspace{1cm} (e) None of the above.

(c) 160 in.$^2$.  \hspace{1cm} $\frac{1}{2} \cdot \frac{\pi}{180} \cdot (4)^2$  \hspace{1cm} $= \frac{\pi}{18}$  \hspace{1cm} $= \frac{8\pi}{9}$ in.$^2$.  \hspace{1cm} $\frac{\pi}{18}$
[33A] Evaluate: \[2 \cos \left( \frac{11\pi}{4} \right) - 4 \sin \left( -\frac{5\pi}{6} \right) - 2 \tan \left( \frac{17\pi}{4} \right)\]

\[2(-\frac{\sqrt{2}}{2}) - 4(-\frac{\sqrt{2}}{2}) - 2(1) = -\sqrt{2} + 2 - 2 = -\sqrt{2}\]

(a) \(4 + \sqrt{2}\)  
(b) \(4 - \sqrt{2}\)  
(d) \(-\sqrt{2}\)  
(e) None of the above.

(c) \(\sqrt{2}\)

[34A] A wheel of radius 4 feet is rotating at 60 rpm (revolutions per minute). What is the linear speed in feet per minute of a point on the circumference of the wheel?

(a) \(480\pi\) ft/min  
(b) \(240\pi\) ft/min  
(c) \(120\pi\) ft/min  
(d) 480 ft/min  
(e) None of the above.

\[\text{Angular speed} = 60 \text{(revolutions per min)} = 120 \text{ radians per min}\]

\[\text{Linear speed} = \sqrt{\text{(Angular speed)}^2} = 480\pi\text{ ft/min}\]

[35A] Find the vertex of the parabola \(5y = x^2 - 8x + 6\).

(a) \((4, -2)\)  
(b) \((-4, 2)\)  
(d) \((4, 2)\)  
(e) None of the above.

(c) \((-4, -2)\)  
\[5y + 16 = (x - 4)^2\]
\[5(y + 2) = (x - 4)^2\] \(\text{(4, -2)}\)

[36A] The graph of \(-4x^2 + 9y^2 - 24x - 90y + 153 = 0\) is:

(a) a circle  
(b) a line  
(c) a parabola  
(d) an ellipse (Not circle)  
(e) a hyperbola

[37A] Find the center of the ellipse \(9x^2 + 4y^2 - 8y - 32 = 0\).

(a) \((1, 0)\)  
(b) \((-1, 0)\)  
(c) \((0, 1)\)  
(d) \((0, -1)\)  
(e) None of the above.

\[9x^2 + 4(y^2 - 2y) = 32\]
\[9x^2 + 4(y^2 - 2y + 1) = 32 + 4\]
\[9x^2 + 4(y - 1)^2 = 40\]

[38A] Simplify \(\frac{\cos^2 \beta}{1 - \sin \beta} - \frac{\cos^2 \beta}{1 + \sin \beta}\).

(a) 0  
(b) 2  
(c) \(2 \sin \beta\)  
(d) \(\cos^2 \beta\)  
(e) None of the above.
[39A] Which is the graph of \( y = -\sin 2x \)?

(a) 

(b) 

(e) 

[40A] Which of the following is the polar graph of \( r = 2 \)?

radius, distance from \( O \) is constant (always 2.)

(a) 

(b) 

(c) 

(d) (circle with center at origin and radius 2.) 

(e)