10. A child claims that \( \frac{1}{2} \) can be greater than \( \frac{1}{3} \) and draws the picture shown to support her argument. How would you respond to this child’s claim? Can you adapt this picture to explain why \( \frac{1}{2} \) is larger than \( \frac{1}{3} \)?

11. Place the following fractions in order from least to greatest, giving careful explanations for how you knew their size relative to each other and benchmark numbers.

\[
\begin{align*}
a. & \quad \frac{4}{5}, \quad \frac{4}{6}, \quad \frac{5}{6}, \quad \frac{2}{5} \\
b. & \quad \frac{11}{13}, \quad \frac{7}{9}, \quad \frac{22}{33}, \quad \frac{5}{7} \\
c. & \quad \frac{31}{25}, \quad \frac{27}{24}, \quad \frac{11}{5}, \quad \frac{23}{20}
\end{align*}
\]

12. There are several strategies that can help you compare fractions. Make a list of strategies you have used or heard used in various activities or whole-class discussions. Give advice to a young child about when to use each of these strategies in ordering fractions. What would you suggest examining first?

13. Describe a mathematical practice in which you engaged in the activities and exercises in this class. Give an explanation with examples.

14. Estimate answers for each of the following fraction computations. Can you say whether your estimate is larger than or smaller than the exact answer would be? Remember what you learned earlier about number neighbors.

\[
\begin{align*}
a. & \quad \frac{5}{8} + \frac{9}{10} \\
b. & \quad 1\frac{7}{16} + 4\frac{3}{27} + 7 \\
c. & \quad 14\frac{9}{16} - 5\frac{1}{16} \\
d. & \quad 4\frac{11}{12} - 2\frac{15}{16} \\
e. & \quad \frac{20}{31} - 1\frac{1}{3} \\
f. & \quad 4\frac{7}{16} + 5\frac{1}{12} - 2\frac{1}{8}
\end{align*}
\]

**Supplementary Learning Exercises for Section 6.2**

1. How do you know that something is incorrect in \( \frac{8}{15} + \frac{6}{11} + \frac{9}{17} = 1\frac{179}{2805} \)?

2. Draw lines from the fractions to the baskets in the figure below. Compare your answers with a classmate’s and discuss any differences.

\[
\begin{array}{cccc}
\frac{5}{7} & \frac{27}{52} & \frac{12}{36} & \frac{32}{36} \\
\frac{11}{24} & \frac{2}{3} & \frac{3}{8} & \frac{40}{45}
\end{array}
\]

3. Find a fraction between each pair of fractional values. Assume that the two fractions refer to the same unit. Justify your reasoning.

\[
\begin{align*}
a. & \quad \frac{2}{3} \text{ and } \frac{5}{6} \\
b. & \quad \frac{5}{8} \text{ and } \frac{7}{9} \\
c. & \quad \frac{27}{30} \text{ and } \frac{40}{38}
\end{align*}
\]

4. Name the benchmark fraction to which each of these fractions is closest.

\[
\begin{align*}
a. & \quad \frac{6}{25} \\
b. & \quad \frac{30}{43} \\
c. & \quad \frac{11}{16} \\
d. & \quad \frac{15}{24} \\
e. & \quad \frac{8}{31}
\end{align*}
\]

5. Describe a way to determine when a fraction is close to \( \frac{2}{3} \).

---

**6.3 Equivalent (Equal) Fractions**

At times fractions that look different can refer to the value of the same quantity. For example, the numerals \( \frac{2}{3} \) and \( \frac{100}{150} \) certainly look different to children, but as you know, these two fractions are **equivalent** or **equal fractions** (elementary school textbooks use both terms). For example, \( \frac{3}{4} = \frac{6}{8} = \frac{9}{12} \) is reasonable to conclude from these drawings because the same amount of the unit is shaded in each drawing.
Chapter 6: Meanings for Fractions

### Example 2

...
What factors do they have in common? A Venn diagram is drawn to show that the two numbers 48 and 64 share four factors of 2. This indicates that both numbers are divisible by 2, 4, 8, and 16. We can also see that since 3 is not a factor shared by both numbers, 64 is not divisible by 3. Furthermore, the largest factor they have in common is 16 (i.e., $2 \cdot 2 \cdot 2 \cdot 2$).

Thus, $\frac{48}{64} = \frac{3 \cdot 16}{4 \cdot 16} = \frac{3}{4} \cdot \frac{16}{16} = \frac{3}{4}$. 

The greatest common factor (GCF or gcd) of two (or more) whole numbers is the largest number that is a factor of the two (or more) numbers. For example, 2, 3, and 6 are all common factors of 12 and 18; 6 is the greatest common factor of 12 and 18. The GCF is sometimes also called the greatest common divisor (GCD or gcd).

In a discrete context, students often have difficulty seeing that two fractions are equivalent. For example, how might you illustrate that $\frac{3}{12} = \frac{1}{4}$ within a discrete context? The same diagram representing one whole should demonstrate 3 parts out of 12 equal parts, while at the same time demonstrating 1 part out of 4 equal parts. An illustration such as the tray of cupcakes here can be used. The given drawing shows 12 cupcakes, with 3 cupcakes having chocolate icing. The same drawing also shows 4 groups (the columns), with 1 group having chocolate icing. So $\frac{3}{12}$ does equal $\frac{1}{4}$.

Often you can visualize a collection of discrete objects in multiple ways. A collection of 12 objects, such as the cupcakes (or perhaps with another arrangement), can be seen as made up of 2, 3, 4, or 6 equal groups. Notice that each of 2, 3, 4, and 6 is a factor of 12.

An important related idea is that of relatively prime numbers. Numbers are relatively prime when they have no common factors except 1. For example, 3 and 5 are relatively prime, as are 3 and 4, even though 4 is not itself prime. Thus, we can say that a fraction is in its lowest terms when the numerator and denominator are relatively prime.

**DISCUSSION 4 Do They Have Common Factors?**

Which of these pairs of numbers is relatively prime? How do you decide?

1. a. 16 and 27  
   b. 75 and 102  
   c. 625 and 835 

As you know, it can be useful to write equivalent fractions so that two fractions have a common denominator, as in $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. The fractions $\frac{8}{12}$ and $\frac{9}{12}$ have a common denominator.

A common denominator can be found for 48 and 64 by finding a number that has both 48 and 64 as a factor. The equation $48 \times 64 = 3072$ certainly works, but smaller numbers do as well. Writing the multiples of each number, we look for the smallest one they have in common.

$48, 96, 144, (192)…$  
$64, 128, (192)…$

The least common multiple (LCM) of two (or more) whole numbers is the smallest number that is a multiple of the two (or more) numbers.

We can also find the least common multiple by looking at the Venn diagram of prime factors. The prime factors they have in common need not be considered twice. In this case, to find the LCM, 64 is multiplied by the 3 that is not a factor of 64. Another way to think about the LCM of these two numbers is to think about the 48 multiplied by the 4 $(2 \times 2)$ that is not a factor of 48.
Number Lines and Equivalent Fractions

This fourth-grade textbook page asks students to show equivalent fractions on a number line. What are good justifications for \( \frac{5}{9} \)?

- The LCM can be found by multiplying 30 by 3 or 45 by 2. The LCM is 90.

The GCF is 3, so 3.

What are the GCF and LCM of 30 and 45? Write each as a product of primes.

\[ 30 = 2 \cdot 3 \cdot 5 \text{ and } 45 = 3 \cdot 3 \cdot 5 \]
6.3 Equivalent (Equal) Fractions

TAKE-AWAY MESSAGE... The ability to recognize equivalent fractions is fundamental to operating with them. The focus in this section was to find a meaning for the methods we used to find equivalent fractions.

Learning Exercises for Section 6.3

1. a. Write 10 fractions that are equivalent to \( \frac{2}{3} \) and 10 fractions that are equivalent to \( \frac{5}{8} \).
   b. Do any of your 20 fractions have common denominators?
   c. When would having common denominators be useful?

2. Use sketches of regions or number lines to show each pair of equivalent fractions:
   a. \( \frac{2}{3} = \frac{10}{15} \)
   b. \( 1\frac{4}{10} = 1\frac{2}{5} \)
   c. \( \frac{5}{4} = \frac{15}{12} \)

3. Find at least one equivalent fraction for each of the following fractions and demonstrate on a number line that the two fractions are equivalent:
   a. \( \frac{3}{5} \)
   b. \( \frac{3}{4} \)
   c. \( \frac{0}{2} \)

4. Suppose a school committee is made up of 6 girls and 4 boys. What fraction describes the part of the committee that is girls? How would you show, through a diagram, that \( \frac{3}{10} \) also describes that part? (Hint: GGGGGGBBBB can be organized as GG GG GG ...)

5. Write each fraction in lowest terms. For efficiency's sake, try to find the greatest common factor of the numerator and denominator.
   a. \( \frac{450}{720} \)
   b. \( \frac{24 \cdot 45 \cdot 17}{64 \cdot 12 \cdot 17} \)
   c. \( \frac{225}{144} \)
   d. \( \frac{x^3 y^5 z^6}{x^4 y^2 z^3} \)
   e. \( \frac{(4.2 \times 10^7) \times (1.5 \times 10^6)}{4.9 \times 10^4} \)

6. For the following pairs, place <, >, or = between the two fractions. Use your knowledge of fraction size to complete this exercise.
   a. \( \frac{3}{4} \)
   b. \( \frac{102}{101} \)
   c. \( \frac{75}{76} \)
   d. \( \frac{8}{9} \)
   e. \( \frac{40}{45} \)
   f. \( \frac{8}{9} \)
   g. \( \frac{9}{10} \)
   h. \( \frac{13}{18} \)
   i. \( \frac{29}{62} \)

7. Using the context of a box of 24 crayons, illustrate that the fractions in each part are indeed equivalent.
   a. \( \frac{4}{24} = \frac{1}{6} \)
   b. \( \frac{18}{24} = \frac{3}{4} \)
   c. \( \frac{4}{12} = \frac{2}{6} \)

8. With the discrete model, choosing the unit to illustrate given equivalent fractions takes some thought. Just any number of objects may not fit the fractions well; for example, a set of 4 objects would not work for showing \( \frac{2}{4} = \frac{3}{6} \). Describe a usable unit for showing that each of the following pairs of fractions is equivalent. Then show each fraction on a separate sketch of the unit, in such a way that the equivalence is visually clear. Make your sketch fit the context.
   a. \( \frac{4}{5} = \frac{8}{10} \)
   b. \( \frac{6}{9} = \frac{4}{6} \)
   c. \( \frac{2}{3} = \frac{4}{6} \)
   d. \( \frac{2}{3} = \frac{12}{18} \)
   e. Context: popsicle sticks
   f. Context: marbles (Do you have the smallest unit?)
   g. Context: children
   h. Context: Choose your own.

9. For illustrating that \( \frac{3}{2} = \frac{9}{12} \), a student repeats a pattern that represents \( \frac{3}{2} \) to show \( \frac{9}{12} \), as in the following diagram. He explains that you can multiply the fraction by 3 to make an equivalent fraction. What problems are there with this diagram and his explanation?

   \[ XXX = XXX XXX XXX XXX O \]

10. Sometimes a textbook or a teacher will refer to reducing a fraction like \( \frac{9}{12} \). What possible danger do you see in that terminology?
11. Each letter on this number line represents a number. Match the numbers to each operation shown without doing any calculation. Use good number sense.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

a. $\frac{3}{4} \times \frac{4}{3}$
b. $2 \times \frac{2}{3}$
c. $\frac{3}{4} \times \frac{1}{2}$
d. $2 + \frac{2}{5}$
e. $\frac{1}{3} \times \frac{2}{5}$

12. In each case, use number sense rather than equivalent fractions to order these fractions and decimals. Use only the symbols $<$ and $=$. Explain your reasoning.

a. $\frac{7}{10}$
b. $\frac{3}{15}$
c. $0.60$
d. $0.4$
e. $0.1$
f. $0.2$

13. Go to http://illuminations.nctm.org/ and search for Equivalent Fractions in the Interactive Activities for Number and Operations. Do several sets of problems, as directed in the instructions, sometimes using circles, sometimes squares.

14. In the same activity given in Exercise 13, use the circle choice. Ignore the red circle; use only the blue and green circles to find which is the larger fraction. Use the $<$, $>$, and $=$ symbols as appropriate. You may need to use the number line below the circles.

a. $\frac{2}{3}$
b. $\frac{3}{5}$
c. $\frac{3}{10}$
d. $\frac{5}{16}$
e. $\frac{7}{11}$
f. $\frac{7}{11}$
g. $\frac{5}{14}$
h. $\frac{5}{16}$

**Supplementary Learning Exercises for Section 6.3**

1. What does it mean to say that three fractions are “equivalent”?

2. a. Show $\frac{2}{3}$ of the pentagonal region illustrated here.
   b. Show that $\frac{2}{3}$ is equivalent to $\frac{8}{15}$ by adding marks to the work for part (a).
   c. How might you show that $\frac{6}{5} = \frac{12}{10}$, with the pentagonal region as the whole?

3. a. Write six fractions that are equivalent to $\frac{2}{3}$.
   b. Which, if any, of your fractions in part (a) is largest?

4. a. Show twelfths on the circular region illustrated here.
   (Hint: Think of a clock, with marks for 12, 3, 6, and 9 first.)
   b. How would you illustrate that $\frac{9}{12} = \frac{3}{4}$, using the circular region?

5. Write the following fractions in simplest form (or lowest terms):

   a. $\frac{3 \times 43}{4 \times 43}$
   b. $\frac{42}{64}$
   c. $\frac{1540}{8800}$
   d. $\frac{105}{140}$
   e. $\frac{288}{1000}$
   f. $\frac{XY}{X^2}$
   g. $\frac{2.4 \times 10^5}{3.6 \times 10^7}$
   h. $\frac{a^2b^6}{a^2b^{12}}$
   i. $\frac{75x}{90}$
   j. $\frac{x^3(x^2 + y^2)}{x^8(x^2 + y^2)}$

6. a. Show and label the points for $\frac{4}{5}$ and $\frac{13}{5}$ on the number line.

   b. Show $\frac{5}{6}$ of this 6-unit-long stick.

   c. Why are the results different in parts (a) and (b)?
7. Use your fraction sense or common denominators to find the larger fraction in each pair. (Try fraction sense first.)
   a. \( \frac{29}{65} \) and \( \frac{163}{190} \)  
   b. \( \frac{29}{65} \) and \( \frac{86}{195} \)  
   c. \( \frac{37}{72} \) and \( \frac{44}{81} \)  
   d. \( \frac{55}{72} \) and \( \frac{83}{108} \)

8. A box contains 12 black marbles and 3 white marbles. What fraction describes the part of the marbles that are black? How would you show, using a diagram of marbles, that \( \frac{4}{5} \) also describes the part made up of black marbles?

9. Given a number line and only the points for the indicated fractions, accurately locate and label the point for the given numbers. If your markings do not make your reasoning clear, add an explanation.

   a. \( \frac{0}{6} \)  
   b. \( \frac{1}{6} \)  
   c. \( \frac{1}{2} \)

10. Locate and label the points for the given numbers, as in Supplementary Learning Exercise 9.

   a. \( \frac{1}{2} \)  
   b. \( \frac{3}{4} \)  
   c. \( \frac{2}{3} \)

### 6.4 Relating Fractions, Decimals, and Percents

When children in a research study were asked to find the sum \( 5 + 0.5 + \frac{1}{2} \), many said it could not be done because they thought fractions and decimals could not be combined. But as you know, fractions and decimals (and percents) are very closely related, even though they look different. All fractions can be expressed as decimals, using a division interpretation of a fraction, and many decimals can be expressed as fractions in a nonzero whole number form. Furthermore, both fractions and decimals can be represented as percents.

**THINK ABOUT …**

In the last chapter you practiced estimating with percents. How are percents related to fractions and decimals?

### ACTIVITY 9 Reviewing Something I Learned Long Ago

Write the following expressions as percents:

a. \( \frac{3}{4} \)  
   b. \( \frac{1}{2} \)  
   c. 0.37  
   d. 1.45  
   e. 0.028  
   f. 43.21

Write the following percents as decimals and as fractions:

g. 32%  
   h. 123%  
   i. 0.01%  
   j. 43.2% 

### ACTIVITY 10 Many Different Meanings

1. Each square grid represents a unit of one whole. Fill in the amount of each square grid indicated by the given number. For each, explain and justify how you know how much to fill in. Be sure you can say what the 4 represents.