7. Use your fraction sense or common denominators to find the larger fraction in each pair. (Try fraction sense first.)
   a. \( \frac{29}{65} \) and \( \frac{163}{190} \)  
   b. \( \frac{29}{65} \) and \( \frac{86}{195} \)  
   c. \( \frac{37}{72} \) and \( \frac{44}{81} \)  
   d. \( \frac{55}{72} \) and \( \frac{83}{108} \)

8. A box contains 12 black marbles and 3 white marbles. What fraction describes the part of the marbles that are black? How would you show, using a diagram of marbles, that \( \frac{4}{5} \) also describes the part made up of black marbles?

9. Given a number line and only the points for the indicated fractions, accurately locate and label the point for the given numbers. If your markings do not make your reasoning clear, add an explanation.
   
   \[ \overbrace{\frac{2}{3} \quad \frac{5}{6}}^{\text{a. } \frac{0}{6}} \quad \overbrace{\frac{1}{2}}^{\text{b. } \frac{1}{6}} \text{ c. } \frac{1}{2} \]

10. Locate and label the points for the given numbers, as in Supplementary Learning Exercise 9.
   
   \[ \overbrace{\frac{11}{12} \quad \frac{11}{13}} \]
   
   a. \( \frac{1}{2} \)  
   b. \( \frac{3}{4} \)  
   c. \( \frac{2}{3} \)

6.4 Relating Fractions, Decimals, and Percents

When children in a research study\(^2\) were asked to find the sum \( 5 + 0.5 + \frac{1}{2} \), many said it could not be done because they thought fractions and decimals could not be combined. But as you know, fractions and decimals (and percents) are very closely related, even though they look different. All fractions can be expressed as decimals, using a division interpretation of a fraction, and many decimals can be expressed as fractions in a nonzero whole number form. Furthermore, both fractions and decimals can be represented as percents.

**THINK ABOUT ...**

In the last chapter you practiced estimating with percents. How are percents related to fractions and decimals?

**ACTIVITY 9** Reviewing Something I Learned Long Ago

Write the following expressions as percents:

a. \( \frac{3}{4} \)  
   b. \( \frac{1}{2} \)  
   c. \( 0.37 \)  
   d. \( 1.45 \)  
   e. \( 0.028 \)  
   f. \( 43.21 \)

Write the following percents as decimals and as fractions:

G. \( 32\% \)  
   H. \( 123\% \)  
   I. \( 0.01\% \)  
   J. \( 43.2\% \)

**ACTIVITY 10** Many Different Meanings

1. Each square grid represents a unit of one whole. Fill in the amount of each square grid indicated by the given number. For each, explain and justify how you know how much to fill in. Be sure you can say what the 4 represents.
When the denominator of a fraction is a power of 10, changing the fraction to a decimal number:

1. Which of these decimals terminate? Write each denominator in each fraction.

- $\frac{1}{2}$
- $\frac{3}{8}$
- $\frac{1}{4}$
- $\frac{2}{3}$
- $\frac{9}{16}$
- $\frac{7}{16}$
- $\frac{4}{9}$
- $\frac{1}{2}$

When decimals did not terminate, can you find a part of each of these decimals that terminate?

2. Which decimals do not terminate? Can you find a part of each of these decimals that terminate?

- $0.3\overline{3}
- 0.1\overline{2}
- 0.0\overline{8}
- 0.16\overline{6}
- 0.3\overline{0}
- 0.25\overline{0}
- 0.03\overline{0}
- 0.\overline{7}$

Activity 11: Can Every Fraction Be Represented as a Decimal?

- The fractional part expressed as a decimal
- The fractional part of the area that is shaded
- The percentage of the area that is shaded
- Using the diagram, explain how to determine

Shade 6 of the small squares in the rectangle.
6.4 Relating Fractions, Decimals, and Percents

EXAMPLE 4
\[
\frac{7}{40} = \frac{7}{2^1 \times 5^2} = \frac{175}{1000} = 0.175
\]

EXAMPLE 5
\[
\frac{42}{175} = \frac{7 \times 6}{7 \times 25} = \frac{6}{25} = \frac{6 \times 2^2}{5^2 \times 2^2} = \frac{24}{100} = 0.24
\]

THINK ABOUT …

The fraction \( \frac{3}{12} \) yielded a terminating decimal representation, but \( \frac{12}{3} \) cannot be factored as a product of only 2s and/or 5s. Why did this happen? Why did \( \frac{3}{12} \) give a terminating decimal?

From Examples 4 and 5 and the preceding Think About, we can make the following conclusion:

A fraction \( \frac{a}{b} \) in lowest terms can be represented with a terminating decimal when the denominator has only 2s and/or 5s as factors, because we can always find an equivalent fraction with a denominator that is a power of 10.

DISCUSSION 5 Remainders That Repeat

1. What are the possible remainders when you divide a whole number by 7? How many different possible remainders are there? Test your conclusion by calculating 1 divided by 7. When a remainder repeats, what happens when you continue the division calculation?

2. How many possible remainders are there when you divide a whole number by 11? Test your conclusion on \( \frac{1}{11} \) and revise your conclusion if necessary.

Nonterminating, repeating decimals, like those you get for \( \frac{1}{7} \) and \( \frac{3}{11} \), are often abbreviated by putting a bar over the repeating part.

EXAMPLE 6

\[
4.333333 \ldots = 4.\overline{3} \quad \text{and} \quad 1.7245245245245 \ldots = 1.\overline{7245}
\]

THINK ABOUT …

How many repeating digits might there be in the decimal equivalent for \( \frac{a}{73} \) for different whole number values for \( n \)?

Focus on SMP MP8

Mathematically proficient students look for and express regularity in repeated reasoning. In this set of activities, you notice when calculations are repeated and, furthermore, can generalize about possible remainders.

We now can conclude one more fact about a fraction in simplest form.

In general, a fraction \( \frac{a}{b} \) in lowest terms can be represented with a nonterminating, repeating decimal if the denominator has factors other than 2s and 5s.
The reverse question can now be considered: can every decimal be represented as a fraction?

Chapter 6: Meanings for Fractions

130
written as a fraction in which the numerator and denominator are whole numbers, and its decimal is nonterminating and nonrepeating. The set of rational numbers together with the set of irrational numbers form the set of **real numbers**. All numbers used in this course are real numbers, and they are sufficient for all of elementary and middle school mathematics. Shown here is a Venn diagram illustrating how some sets of numbers are related.

**THINK ABOUT ...**
\[ \sqrt{3} \text{ on a calculator may give } 1.7320508. \text{ But } 1.7320508 = \frac{17,320,508}{10,000,000} \]
so why is \( \sqrt{3} \) said to be irrational?

**TAKE-AWAY MESSAGE . . .** All fractions of the form \( \frac{a}{b} \), with whole numbers \( a \) and \( b \), where \( b \neq 0 \), can be written in decimal form; some terminate and some repeat. Conversely, decimals that terminate or repeat can be written as fractions. These numbers are known as rational numbers. If a nonterminating decimal does not repeat, it names an irrational number. Together, the rational numbers and the irrational numbers make up the set of numbers we call **real numbers**.

**Learning Exercises for Section 6.4**

1. Determine which of the fractions in parts (a–f) have repeating decimals, and explain why. Then write each fraction as a terminating or repeating decimal.
   a. \( \frac{3}{8} \)  
   b. \( \frac{23}{10} \)  
   c. \( \frac{3}{7} \)  
   d. \( \frac{3}{11} \)  
   e. \( \frac{9}{24} \)  
   f. \( \frac{5}{6} \)  
   g. Two answers are the same. Why?

2. Write the following decimals as fractions, if possible. Change to simplest form or lowest terms.
   a. 0.625  
   b. 0.49  
   c. 91.333 . . .  
   d. 1.7  
   e. \( \frac{9}{9} \)  
   f. 0.93  
   g. 0.53  
   h. 0.053  
   i. 4.76  
   j. 8.094  
   k. 0.1213141516171819110111112113114115 . . .

   [There is a pattern in part (k). Assume that the pattern you see continues. Is there a repeating block of digits?]

3. a. Are there any fractions between \( \frac{7}{15} \) and \( \frac{8}{15} \)? If so, how many?
   b. Are there any decimal numbers between \( \frac{7}{15} \) and \( \frac{8}{15} \)?

4. Find decimal numbers between each pair of numbers [assume the pattern in part (d) continues]:
   a. 0.4 and 0.5  
   b. 0.444 and 0.445  
   c. 1.3567 and 1.356777777 . . .  
   d. 0.0000011111111 . . . and 0.000001100110011 . . .

5. Place the following in order from smallest to largest, without using a calculator:
   \( \frac{2}{3}, \frac{5}{6}, 0.5\overline{6}, 1.23, \frac{3}{17}, \frac{12}{15}, \frac{11}{12}, \frac{29}{24}, 0.2\overline{6}, \frac{1}{4}, 0.21 \)

6. Explain how you know there is an error, without calculating:
   \( \frac{\text{some whole number}}{7} = 4.2923456789 \)

7. One type of calculator can show only eight digits. Suppose the calculator answer to a calculation is 8.1249732. Explain how the exact answer could be the following:
   a. a terminating decimal  
   b. a repeating decimal  
   c. an irrational number
<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/4</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>1/8</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>4/7</td>
<td>0.571</td>
<td>57.1%</td>
</tr>
<tr>
<td>220/100</td>
<td>2.2</td>
<td>220%</td>
</tr>
<tr>
<td>7/13</td>
<td>0.538</td>
<td>53.8%</td>
</tr>
<tr>
<td>3/1</td>
<td>3.0</td>
<td>300%</td>
</tr>
<tr>
<td>10/11</td>
<td>0.909</td>
<td>90.9%</td>
</tr>
</tbody>
</table>

13. To help understand fractions, draw a circle to represent the following fraction.

12. A student says, "My calculator shows that 2/3 is 0.4142136 and 1.4142136 = 1 1/2."
This mean that 1 1/2 is equivalent to a terminating decimal. Explain.

11. A student says, "My calculator shows 0.0709231 for 1/14 when I divide 1 by 14."
Does this mean that 1/14 is equivalent to a terminating decimal? Explain.

10. Chris says, "0.3 is bigger than 0.5 because 0.7 is bigger than 0.3."
San says, "0.7 is bigger than 0.3 because 0.7 is bigger than 0.3.

9. Complete the following table:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10</td>
<td>0.1</td>
<td>10%</td>
</tr>
<tr>
<td>1/20</td>
<td>0.05</td>
<td>5%</td>
</tr>
<tr>
<td>1/40</td>
<td>0.025</td>
<td>2.5%</td>
</tr>
<tr>
<td>1/80</td>
<td>0.0125</td>
<td>1.25%</td>
</tr>
<tr>
<td>1/100</td>
<td>0.01</td>
<td>1%</td>
</tr>
<tr>
<td>1/400</td>
<td>0.0025</td>
<td>0.25%</td>
</tr>
<tr>
<td>1/1000</td>
<td>0.001</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

8. Draw lines to show which of these decimal numbers and percentages are closest to:

Chapter 6: Meanings for Fractions

132
14. Complete the following table:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>50/54</td>
<td></td>
<td>500%</td>
</tr>
<tr>
<td>95/67</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>73%</td>
</tr>
<tr>
<td>1/3</td>
<td></td>
<td>235%</td>
</tr>
</tbody>
</table>

15. a. About what percent of the circle shown here does each piece represent?

A _____ B _____ C _____ D _____ E _____

What percent is the sum of your estimates?

Is this reasonable?

b. About what fraction of the circle does each piece represent?

A _____ B _____ C _____ D _____ E _____

What should the sum of these fractional parts be?

16. There are some commonly used fractions for which you should know the decimal equivalent without needing to divide to find it. And, of course, if you know decimal equivalents, it is quite easy to find percent equivalents. Give all the decimal and percent equivalents you know to the following fractions. Find the others by dividing, and then memorize those, too.

a. 1/8  b. 1/5  c. 1/4  d. 1/3  e. 2/5  f. 3/8
  g. 5/4  h. 1/2  i. 3/5  j. 5/8  k. 2/3  l. 3/4
  m. 4/5  n. 7/8  o. 1  p. 7/4

17. Name a fraction or whole number that is close to being equivalent to the following percentages:

a. 24%  b. 95%  c. 102%  d. 35%  e. 465%  f. 12%

18. In Chapter 5 you learned different ways of finding fractional equivalents for percents. For example, 10% = \( \frac{1}{10} \), 25% = \( \frac{1}{4} \), and 50% = \( \frac{1}{2} \). Use this knowledge to estimate each of the following amounts:

a. 15% of 798  b. 90% of 152  c. 128% of 56
  d. 65% of 24  e. 59% of 720  f. 9% of 59
  g. 148% of 32  h. 0.1% of 24  i. 32% of 69

19. Draw a diagram that shows why \( \frac{3}{4} \) of a given amount is the same as \( \frac{8}{10} \) of the same amount.
11. From memory, give familiar fractions that are roughly equal to the following.

10. What is a good estimate of \( \frac{11}{9} + \frac{3}{4} + 0.6 \) and why?

\[
\frac{2}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}, \frac{11}{11}
\]

10. What is a good estimate of \( \frac{11}{9} + \frac{3}{4} + 0.6 \) and why?

Fractions:

Show how the decimals and percents could be determined for the following.

9. The decimal 3.1416 is sometimes used for the number \( \pi \) which is irrational.

8. The decimal 0.0499499499... is sometimes used for the number \( e \) which is irrational.

7. If possible, give two decimals between each pair. If not possible, explain why.

6. Why do you say that is irrational? What would be

5. Is irrational? Why do you say that is irrational? What would be

4. Write whole numbers rational or irrational? Explain.

3. In terms of decimals, what is a rational number? An irrational number? A real number?

2. Write these decimals as (exact) fractions of the form

\[
\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \frac{g}{h}, \frac{i}{j}, \frac{k}{l}, \frac{m}{n}, \frac{o}{p}
\]

1. Which of the following numbers will have terminating decimals? Explain how you know.
15. a. The accompanying diagram is split into several regions. Each region is about what percent of the entire figure?

A ____ B ____ C ____ D ____ E ____ F ____

b. Each region is about what fraction of the entire figure?

A ____ B ____ C ____ D ____ E ____ F ____

16. News report “The water level in the river is 3.8 feet above flood stage. We expect the water level to decrease about a foot by tomorrow.” The anticipated decrease will be about what percent of flood water?

17. Sketch a line segment about 68% longer than this one:

18. In an election with only two candidates, A and B, Candidate A won by 18%. What percentage of the whole vote did each candidate receive?

6.5 Issues for Learning: Understanding Fractions and Decimals

Even young children in the early grades have some understanding of parts and wholes, but they do not always relate this to the fraction symbol. There are several critical ideas that children need to understand before they can successfully operate on fractions:

1. When breaking a whole (in this case a continuous whole) into equal parts, the more parts there are, the smaller each part will be. Thus, \( \frac{1}{3} \) is smaller than \( \frac{1}{2} \). Even though children might know this in a particular setting (e.g., pieces of pizza), they still need to learn it symbolically.

2. Fractions represent specific numbers. That is, \( \frac{3}{2} \) itself is the value of a quantity; \( \frac{3}{2} \) is one number, not two numbers separated by a bar. When students do not understand this relationship between the numerator and the denominator, they operate on them separately. For example, they might say that \( \frac{3}{2} + \frac{2}{3} = \frac{5}{5} \).

3. Equivalence is one of the most crucial ideas students must understand before operating on fractions and decimals. Children will not be able to add or subtract fractions until they can identify and generate equivalent fractions, because they often need to find an equivalent form of a fraction before they can add or subtract.

4. Children must understand why like denominators are necessary before they can add or subtract fractions. Only when there is a common denominator for two fractions are they adding or subtracting like pieces, because now both fractions refer to a common size for the pieces.

Similar problems are encountered when working with decimal notation. Students may not understand what the decimal point indicates. If they estimate the value of 48.85 as, for example, 50.9, then they are treating the 48 separately from the .85.

The work of Chris, Sam, and Lars shown in the Learning Exercises for Section 6.4 illustrates some common misunderstandings of decimals that have been found in a number of countries. Some children think that because hundredths are smaller than tenths, 2.34, which has hundredths, is smaller than 2.3, which has tenths. Others believe that the number of digits indicates relative size, as is true for whole numbers. Thus, 2.34 is considered larger than 2.4 because 234 is larger than 24.

Teachers often have students compare numbers such as 2.34 and 2.3 by “adding zeros until each number has the same number of places.” Now the comparison is between 2.34 and 2.30. Although this technique works, often it is not meaningful and becomes just...