

Math 210

1. Solve the system:

$$\begin{aligned}x + y + z &= 1 \\2x + 3y + 4z &= 5\end{aligned}$$

- (a) $z = 2, y = -1$ and $x = 0$
- (b) $z =$ any value, $y = 3 - 2z$ and $x = z - 2$
- (c) $z =$ any value, $y = 3 - 2z$ and $x = z + 2$
- (d) $z =$ any value, $y = 3 + 2z$ and $x = z - 2$
- (e) None of the above

2. Solve the system:

$$\begin{aligned}2x + 3y - 5z &= -14 \\3x - 2y + 3z &= 17 \\4x + 3y - 2z &= -1\end{aligned}$$

In the solution $x =$:

- (a) 4
- (b) 3
- (c) 1
- (d) 2
- (e) The system is inconsistent.

3. Solve the system:

$$\begin{aligned}5x - 2y - 2z &= 5 \\2x - y - z &= 1 \\-3x + y + 2z &= 2\end{aligned}$$

- (a) In the solution $x = 4$
- (b) In the solution $x = 2$
- (c) In the solution $x = 1$
- (d) In the solution $x = 3$
- (e) None of the above

4. Find the bottom left entry of:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

- (a) 22
- (b) 48
- (c) 43
- (d) 64
- (e) 19

5. Solve the system:

$$\begin{aligned} x + y - 2z &= 3 \\ 2x - 3y + 3z &= 2 \\ 5x - 5y + 4z &= 6 \end{aligned}$$

- (a) In the solution $x = 4$.
- (b) In the solution $x = 2$.
- (c) In the solution $x = 3$.
- (d) In the solution $x = 1$.
- (e) The system is inconsistent.

6. Find the inverse of

$$A = \begin{pmatrix} 0 & 0 & 4 \\ 2 & 0 & 10 \\ 0 & 4 & 8 \end{pmatrix}$$

The entry in the first row and first column of A^{-1} is:

- (a) The matrix A is not invertible.
- (b) $1/2$.
- (c) $-5/4$.
- (d) 0.
- (e) $3/4$.

7. Which of the following systems have infinitely many solutions?

$$I. \begin{cases} x - y + 8z = 7 \\ y - 8z = 3 \end{cases} \quad II. \begin{cases} x - y + 8z + 3w = 10 \\ y - 8z = 3 \\ w = 1 \end{cases}$$

$$III. \begin{cases} x - y + 8z = 7 \\ y - 8z = 3 \\ y - 8z = 3 \end{cases} \quad IV. \begin{cases} x - y + 5z = 7 \\ y - 8z = 3 \\ y - 8z = 4 \end{cases}$$

- (a) I, II, and IV only.
- (b) IV only.
- (c) I, II, and III only.
- (d) I and II only.
- (e) None of the above.

8. Invert the following matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 4 \\ 3 & -3 & -5 \end{pmatrix}$$

In the inverted matrix, the top right entry is:

- (a) 65
- (b) 80
- (c) 5
- (d) -30
- (e) -5

9. Find the bottom left entry of:

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}$$

- (a) 23
- (b) 19
- (c) 34
- (d) 64
- (e) 48

10. Which of the following matrices are invertible?

$$A = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

- (a) A and D only.
- (b) A only.
- (c) A and B only.
- (d) All except C.
- (e) Some other selection.

11.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 10 & 20 \\ 3 & 7 & 14 & 21 \\ 4 & 9 & 17 & 26 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} -35 & -7 & -30 & 35 \\ 26 & 5 & 20 & -24 \\ -4 & -1 & -2 & 3 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

Solve the system:

$$\begin{aligned} x + 2y + 3z + 4w &= 0 \\ 2x + 5y + 10z + 20w &= 1 \\ 3x + 7y + 14z + 21w &= 2 \\ 4x + 9y + 17z + 26w &= 3 \end{aligned}$$

In the solution:

- (a) $w=0$.
 - (b) $w=1$.
 - (c) $w=3$.
 - (d) $w=2$.
 - (e) None of the above.
12. Assume that we have a simple economy with only two industries Gas and Electricity. To produce 1 unit of Gas requires .1 units Gas and .1 units of Electricity. To produce 1 unit of Electricity requires .4 units Gas and .4 units of Electricity. There is a consumer demand for 25 units of Gas and 15 units of Electricity.
- How many units of gas should be produced?
- (a) 57
 - (b) 42
 - (c) 37
 - (d) 25
 - (e) 30

13. A simple economy has 2 industries: oil and gas. The production of \$1 of oil requires \$0.10 in oil and \$0.30 in gas, while the production of \$1 of gas requires \$0.20 of oil and \$0.40 of gas. Assume there is an external demand for \$240 of oil and \$480 of gas.

How much oil should be produced to satisfy the demand?

- (a) \$1050.
- (b) \$500.
- (c) \$750.
- (d) \$1300.
- (e) None of the above.

14.

$$\text{If } A = \begin{pmatrix} 1 & -2 & -3 \\ 0 & -6 & -5 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -2 & 7 \\ 0 & 0 & 9 \end{pmatrix}$$

then the top right entry of $A + B$ is:

- (a) 1.
 - (b) 3.
 - (c) 0.
 - (d) 4.
 - (e) 2.
15. In solving a system of linear equations using Gauss-Jordan elimination, the following augmented matrix is obtained:

$$\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & -3 & 6 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{array}$$

Which is true?

- (a) There are exactly two solutions for this problem.
- (b) There are infinitely many solutions, one of which is $x = 0$, $y = 11$, $z = 2$.
- (c) There is exactly one solution and $x = 6$, $y = 7$, and $z = 0$.
- (d) There are infinitely many solutions, one of which is $x = 9$, $y = 5$, $z = 1$.
- (e) There are no solutions for this problem.

16. Solve the system :

$$\begin{aligned}x - y + 2z &= 14 \\y - 2z &= -7 \\y + 2z &= 33\end{aligned}$$

- (a) In the solution: $x = 1$.
- (b) There are infinitely many solutions.
- (c) No solution; the equations are inconsistent.
- (d) In the solution: $x = -6$.
- (e) In the solution: $x = 7$.

17. Suppose that the matrix product BAB is defined, and is 19×7 . Then:

- (a) The size of B is 7×7 .
- (b) The size of B is impossible to determine.
- (c) BAB can never have size 7×19 .
- (d) The size of B is 7×19 .
- (e) The size of B is 19×7 .

18. Solve the system:

$$\begin{aligned}x + 2y + z &= 8 \\3x + 7y + 4z &= 29 \\3x - 3y - 5z &= -18\end{aligned}$$

- (a) In the solution $x = 3$.
- (b) In the solution $x = 2$.
- (c) In the solution $x = 4$.
- (d) In the solution $x = 1$.
- (e) The system is inconsistent.

19. Given that $\begin{bmatrix} 1 & 0 & 4 & 0 \\ 5 & 1 & 8 & -2 \\ -6 & -2 & 1 & 0 \\ -1 & 0 & -4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -8 & -4 & -16 \\ -3 & 25 & 12 & 50 \\ 0 & 2 & 1 & 4 \\ 1 & 0 & 0 & 1 \end{bmatrix}$, what is the z value in the solution of

$$\begin{aligned} x &+ 4z & & = 8 \\ 5x &+y &+8z &-2w = 7 \\ -6x &-2y &+z & = 4 \\ -x & &-4z &+w = -2 \end{aligned}$$

- (a) 6
 (b) 4
 (c) -14
 (d) 10
 (e) No solution exists.
20. Assume that we have a simple economy with only two industries Gas and Electricity. To produce 1 unit of Gas requires .52 units Gas and .44 units of Electricity. To produce 1 unit of Electricity requires .25 units Gas and .25 units of Electricity. There is a consumer demand for 32 units of Gas and 54 units of Electricity.
- How many units of gas should be produced?
- (a) 160
 (b) 250
 (c) 128
 (d) 32
 (e) 150
21. Which of the following matrices are invertible?

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

- (a) A and D only.
 (b) A only
 (c) A and B only.
 (d) All except C.
 (e) None.

22. Solve the system :

$$\begin{aligned}x + 3y - z &= 1 \\x + 4y - z &= 4 \\2x + 7y - 2z &= 5\end{aligned}$$

- (a) There are infinitely many solutions.
- (b) In the solution: $y = 2$.
- (c) No solution; the equations are inconsistent.
- (d) In the solution: $y = 0$.
- (e) In the solution: $y = 1$.

23. Find the inverse of

$$B = \begin{pmatrix} -35 & -7 & -30 & 35 \\ 26 & 5 & 20 & -24 \\ -4 & -1 & -2 & 3 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

The top left entry of B^{-1} is:

- (a) 2
 - (b) 3
 - (c) 1
 - (d) 4
 - (e) None of the above.
24. A simple economy has 2 industries: oil and gas. The production of \$1 of oil requires \$0.10 in oil and \$0.30 in gas, while the production of \$1 of gas requires \$0.20 of oil and \$0.40 of gas. Assume there is an external demand for \$240 of oil and \$480 of gas.
- How much gas should be produced to satisfy the demand?
- (a) \$1050.
 - (b) \$1300.
 - (c) \$500.
 - (d) \$750.
 - (e) None of the above.

25. Which of the following matrices are invertible?

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 \\ 2 & 4 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

- (a) A and B only.
- (b) A only.
- (c) B only.
- (d) All except C.
- (e) All of them.

26. If $\begin{bmatrix} 4 & 12 \\ 6 & x \end{bmatrix}$ is not invertible, what is x ?

- (a) 6
- (b) 15
- (c) 12
- (d) 18
- (e) 20

27. Solve the system :

$$\begin{aligned}x - 2y + 5z &= -1 \\2x + 3y - 5z &= 5 \\3x + 2y + 5z &= 5\end{aligned}$$

In the solution:

- (a) $z = 2$.
- (b) $z = 1$.
- (c) $z = 0$.
- (d) There are infinitely many solutions.
- (e) No solution; the equations are inconsistent.

28. Assume that we have a simple economy with only two industries Gas and Electricity. To produce 1 unit of Gas requires .1 units Gas and .1 units of Electricity. To produce 1 unit of Electricity requires .4 units Gas and .4 units of Electricity. There is a consumer demand for 25 units of Gas and 15 units of Electricity.

How many units of electricity should be produced?

- (a) 15
 - (b) 27
 - (c) 32
 - (d) 23
 - (e) 45
29. Let A be a 4×3 matrix, B a 3×5 matrix, and C a 4×5 matrix. Which of the following is defined?
- (a) $CB^T + A^T$
 - (b) $A^T C + B$
 - (c) All of the above.
 - (d) $AB + C^T$
 - (e) None of the above.

30.

$$\text{If } A = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 3 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -2 \\ 2 & 1 \\ 0 & 0 \end{pmatrix}$$

then the top right entry of AB is:

- (a) 0
- (b) 1
- (c) 3
- (d) 2
- (e) -1

31. Find the top left entry of:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 & 10 \end{pmatrix}$$

- (a) 7
- (b) 5
- (c) 1
- (d) 3
- (e) None of the above.

32. Find the inverse of

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 5 & 6 \\ 1 & 5 & 11 \end{pmatrix}$$

Which of the following is true?

- (a) The top right entry of the inverse is 0.
 - (b) The top left entry of the inverse is 1.
 - (c) The matrix has more than one inverse.
 - (d) The matrix is not invertible.
 - (e) The bottom right entry of the inverse is $1/11$.
33. A simple economy has 2 industries: oil and gas. The production of \$1 of oil requires \$0.10 in oil and \$0.20 cents in gas, while the production of \$1 of gas requires \$0.30 cents of oil and \$0.40 cents of gas. Assume there is an external demand for \$960 of oil and \$1920 of gas.
- How much oil should be produced to satisfy the demand?
- (a) \$2000.
 - (b) \$4000
 - (c) \$1200.
 - (d) \$1000.
 - (e) None of the above.

34. Invert the following matrix and then answer the question.

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & -3 \\ 4 & 10 & -7 \end{pmatrix}$$

In the inverted matrix, the entry in the middle row and middle column is:

- (a) 3
- (b) 0
- (c) 2
- (d) 1
- (e) None of the above

35. Invert the following matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 4 \\ 3 & -3 & -5 \end{pmatrix}$$

In the inverted matrix, the top right entry is:

- (a) 4
- (b) -4
- (c) 0
- (d) -1
- (e) 1

36. Let $A = \begin{bmatrix} 2 & -1 \\ 6 & -2 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -2 & 4 \end{bmatrix}$. The entry in the first row, second column of BA is

- (a) 0
- (b) 20
- (c) -24
- (d) 4
- (e) 3

37. Which of the following systems have infinitely many solutions?

$$I. \begin{cases} x - y + 5z = 10 \\ x - y = 3 \end{cases} \quad II. \begin{cases} x - y + 5z + 3w = 13 \\ x - y = 3 \\ w = 1 \end{cases}$$

$$III. \begin{cases} x - y + 5z = 10 \\ x - y = 3 \\ x - y = 3 \end{cases} \quad IV. \begin{cases} x - y + 5z = 10 \\ x - y = 3 \\ -x + y = 3 \end{cases}$$

- (a) I, II, III, and IV.
(b) I, II, and III only.
(c) I and II only.
(d) III and IV only.
(e) IV only.
38. Assume that we have a simple economy with only two industries Gas and Electricity. To produce 1 unit of Gas requires .10 units Gas and .14 units of Electricity. To produce 1 unit of Electricity requires .50 units Gas and .70 units of Electricity. There is a consumer demand for 100 units of Gas and 200 units of Electricity. How many units of gas should be produced?
- (a) 600.
(b) 650.
(c) 500.
(d) 2500.
(e) 970.

39. Solve the system :

$$\begin{aligned} x - y + 5z &= 13 \\ y - 2z &= -7 \\ y + 8z &= 33 \end{aligned}$$

- (a) In the solution: $x = -3$.
(b) No solution; the equations are inconsistent.
(c) In the solution: $x = 1$.
(d) In the solution: $x = -6$.
(e) There are infinitely many solutions.

40. Invert the following matrix and then answer the question.

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & -3 \\ 4 & 10 & -7 \end{pmatrix}$$

In the inverted matrix the entry in the top row and middle column is:

- (a) -2
- (b) -1
- (c) -3
- (d) -4
- (e) None of the above

41. Find the inverse of

$$A = \begin{pmatrix} 0 & 4 & 12 \\ 1 & 0 & 3 \\ 0 & 2 & 4 \end{pmatrix}$$

- (a) 1.
- (b) The entry in the second row and second column of A^{-1} is: 0.
- (c) The matrix A is not invertible.
- (d) $1/2$.
- (e) $-1/2$.

42. Consider the system:

$$\begin{aligned}x + 2y + 3z + 5w &= 0 \\x + 3y + 8z + 4w &= -1 \\2x + 5y + 12z + 25w &= -2 \\4x + 10y + 23z + 35w &= 3\end{aligned}$$

Given:

$$A = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 1 & 3 & 8 & 4 \\ 2 & 5 & 12 & 25 \\ 4 & 10 & 23 & 35 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 115 & 110 & 126 & -119 \\ -77 & -75 & -86 & 81 \\ 15 & 15 & 17 & -16 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

In the solution:

- (a) $w=2$.
- (b) $w=0$.
- (c) $w=1$.
- (d) $w=3$.
- (e) None of the above.

43. Find the inverse of

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 5 & 6 \\ 0 & 5 & 7 \end{pmatrix}$$

The entry in the third row and third column is:

- (a) $1/7$.
- (b) 0 .
- (c) -1 .
- (d) 1 .
- (e) None of the above.

44. Solve the system :

$$\begin{aligned}x + y + 10z &= 12 \\2x + 3y - 5z &= 5 \\3x + 2y + 5z &= 5\end{aligned}$$

In the solution:

- (a) There are infinitely many solutions.
- (b) $z = 0$.
- (c) $z = 1$.
- (d) No solution; the equations are inconsistent.
- (e) $z = 2$.

45. Invert the following matrix

$$\begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 1 \\ 4 & -1 & -4 \end{pmatrix}$$

In the inverted matrix the top right entry is?

- (a) 4
- (b) 0
- (c) -4
- (d) -1
- (e) 1

46. Solve the system:

$$\begin{aligned}x + y - 2z &= 3 \\2x - 3y + 3z &= 2 \\5x - 10y + 11z &= 7\end{aligned}$$

In the solution $x =$:

- (a) 4
- (b) 2
- (c) 3
- (d) 1
- (e) The system is inconsistent.

47. Suppose that the matrix product ABC is defined where A is 2×5 and C is 7×9 . Then B is :

- (a) 5×7 .
- (b) 5×9 .
- (c) 7×5 .
- (d) 2×9 .
- (e) 9×2 .

48. A simple economy has 2 industries: oil and gas. The production of \$1 of oil requires \$0.10 in oil and \$0.20 cents in gas, while the production of \$1 of gas requires \$0.30 cents of oil and \$0.40 cents of gas. Assume there is an external demand for \$960 of oil and \$1920 of gas.

How much gas should be produced to satisfy the demand?

- (a) \$1200.
- (b) \$4000
- (c) \$2000.
- (d) \$1000.
- (e) None of the above.

bddceccdc - 10
bbbddeedde - 20
bacacdceba - 30
bdeaebbdd - 40
bedceab