

Make-up Final Exam
Math 211

1. **(25 pts.)** Find the derivative of each of the following functions: (Do not simplify)

(a) $f(x) = \frac{3}{x^5} - \sqrt{3x - 1}$

(b) $f(x) = e^x (x^5 + 2x^3)^3$

(c) $f(x) = (2x^3 + e^{-5x+2})^5$

(d) $f(x) = \left(\frac{1}{x} + x\right) \cdot \ln(5 - 2x^2)$

(e) $f(x) = \frac{2x^2 + 3x}{x^3 + 2}$

2. (20 pts.) Find the following limits algebraically (if exists)

$$(a) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}$$

$$(b) \lim_{x \rightarrow 0} \frac{2x^3 + 3x - 10}{x^4 + x^2 + 5}$$

$$(c) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

$$(d) \lim_{x \rightarrow \infty} \frac{2x^3 - 3x^2 + 2}{3x^3 + 2x - 1}$$

3. (10 pts.) Find the equation of the tangent line to the graph of $f(x) = 5x^3 + 2$ at $x = 2$.

4. **(20 pts.)** Let $f(x) = \frac{4x^2 + 4}{x^2 - 1}$, $f'(x) = \frac{-16x}{(x^2 - 1)^2}$ and $f''(x) = \frac{16(1 + 3x^2)}{(x^2 - 1)^3}$

(a) Find horizontal asymptotes of $f(x)$.

(b) Find vertical asymptotes of $f(x)$.

(c) Find all the intervals where $f(x)$ is increasing and all the intervals where $f(x)$ is decreasing

(d) Find all the intervals where $f(x)$ is concave up and all the intervals where $f(x)$ is concave down.

5. **(15 pts.)** Find the absolute maximum and minimum of the function $f(x) = x + \frac{16}{x}$ on the interval $[1,5]$.

6. **(15 pts.)** A construction company is constructing a closed top, square -based, rectangular metal tank that will have a volume 64 cubic ft. What dimensions yield minimum surface area?

7. **(15 pts.)** A certain bacteria culture grows at a rate proportional to its size and it becomes 4 times its size in every 6 hours. Find the growth rate.

8. **(10 pts.)** The decay rate of zirconium is 1.05% per day. What is its half life?

9. **(15 pts.)** An airconditioning company determines that the marginal cost of producing the x th airconditioner is given by

$$C'(x) = -.2x + 500,$$

and that $C(0) = 100$.

Find the total cost of producing 100 airconditioners.

10. (20 pts.) Evaluate the following integrals:

$$(a) \int \left(\frac{2}{x^3} + \sqrt{x} - e^{-2x} + \frac{3}{x} \right) dx$$

$$(b) \int \frac{(x+1)}{(x^2+2x)} dx$$

$$(c) \int x e^{3x^2+1} dx$$

$$(d) \int_1^5 e^{5x} dx$$

11. (10 pts.) Let the cost of producing x units be

$$C(x) = 20x e^{-.01x} \text{ dollars.}$$

and the revenue from the sale of x units be

$$R(x) = 30x e^{-.01x}.$$

(a) Find the profit function.

(b) What production level will maximize the profit?

12. **(15 pts.)** Find the area of the region bounded by

$$y = 2x^2 - x + 3 \text{ and } y = x + 3.$$

13. (10 pts.) Using the limit definition of derivative, find $f'(x)$ where $f(x) = \frac{1}{2x}$.