

Math 211

Final Exam

FALL 2008

Name (print): _____

Signature: _____

Z-number: _____

Section: _____

Show all work for credit!

Page 1	/ 20
Page 2	/ 15
Page 3	/ 25
Page 4	/ 35
Page 5	/ 25
Page 6	/ 25
Page 7	/ 20
Page 8	/ 20
Page 9	/ 20
Page 10	/ 20

TOTAL	/ 200
--------------	--------------

1. **[20 pts.]** Find the derivative of each of the following functions. (Do not simplify).

(a) $f(x) = \sqrt{\frac{1-2x}{1+2x}}$

(b) $f(x) = \ln\left(\frac{x^2+5}{2x+1}\right)^2$

(c) $f(x) = e^{x^2-3x+5} \cdot \ln(5-x)$

(d) $f(x) = e^{3x} \ln x + (\ln x)^2 + \ln 2x$

2. **[15 pts.]** Find the following limits (if they exist):

(a) $\lim_{h \rightarrow 0} \frac{-5}{x(x+h)}$

(b) $\lim_{x \rightarrow \infty} \frac{x^3 - 5x^2 + 8}{4x^3 - 2x + 9}$

(c) Consider

$$f(x) = \begin{cases} -4 & \text{for } x = 3 \\ 2x + 3 & \text{for } x \neq 3 \end{cases}$$

i. Compute $\lim_{x \rightarrow 3} f(x)$, if it exists.

ii. Is $f(x)$ continuous at $x = 3$? Give reasons for your answers.

3. **[10 pts.]** Let $f(x) = 1 + e^{-2x}$. Find the equation of the tangent line at $x = 0$.

4. **[15 pts.]** Find the absolute maximum and minimum values of $f(x) = x^3 - x^2 - x - 2$ on the interval $[0, 2]$.

5. **[15 pts.]** The population of the United States was 280 million in the year 2000. Suppose population $P(t)$ was growing exponentially at the rate of .9% per year.

(a) After what period of time will the population be triple that in 2000?

(b) What will be the population of the United States in 2015?

6. **[20 pts.]** Using the limit definition of the derivative of a function, find $f'(x)$ where

$$f(x) = x^2 - x.$$

7. **[25 pts.]** Let $f(x) = x^3 - 3x + 2$.

(a) Determine $f'(x)$ and $f''(x)$.

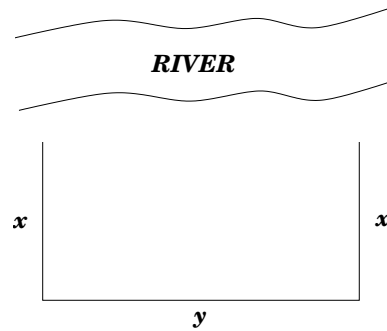
(b) Find the critical values (points).

(c) Find the relative maxima/minima of the function using a first derivative test.

(d) Use the second derivative test to determine where the graph of $f(x)$ is concave upward **and** concave downward and find the inflection point(s).

(e) Draw the rough sketch of the graph of $f(x)$ using the information in (b)-(d).

8. **[20 pts.]** A rancher wants to build a rectangular ranch next to a river fencing off on three sides (as shown below), using 120 yd of fencing. What dimensions of the rectangle will produce maximum area? What is the maximum area?



9. **[20 pts.]** A store manager determines that in order to sell x units of computer, the price per unit must be

$$p = 1000 - x.$$

The total cost of producing x units is given by

$$C(x) = 3000 + 2x.$$

- (a) Find the total profit, $P(x)$.

- (b) How many units must be produced and sold, so that the profit will be maximum?

- (c) What is the maximum profit?

10. **[20 pts.]** Compute the following integrals:

$$(a) \int (7e^{6x} - \sqrt{x}) \, dx$$

$$(b) \int \left(1 - \frac{3}{x} + \frac{1}{x^4}\right) \, dx$$

$$(c) \int x^2(x^3 + 1)^{10} \, dx$$

$$(d) \int_1^e \left(\frac{1}{x} + 5\right) \, dx$$

11. **[20 pts.]** Find the area of the region bounded by the graphs of $y = 2x + 1$ and $y = x^2 + 1$.