

Final Exam

Math 211

SPRING 2008

Name (print): _____

Signature: _____

Z-number: _____

Section: _____

Show all work for credit!

Page 1	/ 20	<table border="1"><tr><td>TOTAL</td><td>/ 200</td></tr></table>	TOTAL	/ 200
TOTAL	/ 200			
Page 2	/ 25			
Page 3	/ 25			
Page 4	/ 30			
Page 5	/ 25			
Page 6	/ 25			
Page 7	/ 40			
Page 8	/ 20			
Page 9	/ 15			

1. **[20 pts.]** Find the derivative of each of the following functions. (Do not simplify).

$$(a) f(x) = \left(5x^2 - \frac{1}{x} + 2x^3\right)^3$$

$$(b) f(x) = \left(\ln\left(\frac{x^2 - 2x}{x^2 + 2x}\right)\right)^5$$

$$(c) f(x) = \left(\frac{x + 1}{x - 1}\right)^3$$

$$(d) f(x) = x\sqrt{x} + e^{x^2-3x} + 3$$

2. **[15 pts.]** Find the following limits (if they exist):

$$(a) \lim_{x \rightarrow 1} \frac{(x-1)}{(x^2 - 3x + 2)}$$

$$(b) \lim_{x \rightarrow 0} (16x^3 + 9x + 25)^{1/2}$$

$$(c) \lim_{x \rightarrow \infty} \frac{2x^3 + 3x^2 + 2}{8x^3 - 3x + 5}$$

3. **[10 pts.]** Let $f(x) = xe^{-x}$. Find the equation of the tangent line at $x = 1$.

4. **[15 pts.]** Let $f(x) = \frac{1}{3}x^3 - 4x$. Find the absolute maximum and minimum values of $f(x)$ on the interval $[-1, 3]$.

5. **[10 pts.]** A bank advertises that it compounds interest continuously and that it will double your money in 15 yrs.

(a) What is its annual interest rate?

(b) Find the function $P(t)$ (balance after t years) where P_0 is the initial investment.

6. **[15 pts.]** Strontium - 90 has a decay rate of 2.8% per year. Suppose 2000 gms of strontium - 90 is present at $t = 0$. (a) How much will remain after 10 yrs? (b) After how long will half of the 2000 gms remain?

7. **[15 pts.]** Using the **limit definition** of the derivative of a function, find $f'(x)$ where

$$f(x) = \sqrt{x+1}$$

8. **[25 pts.]** Let $f(x) = 2x^3 - 9x^2 + 12x$.

(a) Determine $f'(x)$ and $f''(x)$.

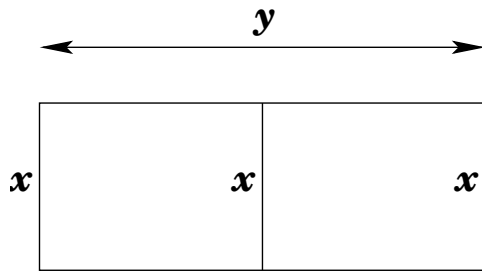
(b) Find the critical points (values).

(c) Find the relative maxima/minima of the function using the first derivative test.

(d) Use the second derivative to determine where the graph of $f(x)$ is concave upward and concave downward and find the inflection point(s).

(e) Draw the rough sketch of the graph of $f(x)$ using the information in (b)-(d).

9. **[20 pts.]** A rectangular corral of 54 sq. ft is to be fenced off and then divided by a fence into two sections, as shown below. Find the dimensions of the corral so that the amount of fencing required is minimized.



10. **[20 pts.]** An appliance company estimates that its daily cost function is $C(x) = 4000 - 400x$, and its total revenue function is $R(x) = 600x - 20x^2 + 8000$. Find the value of x that maximizes the total profit.

11. **[10 pts.]** Compute the following integrals:

$$(a) \int \left(x^2 + 3x - \frac{1}{x} + 2e^{5x} \right) dx$$

$$(b) \int x e^{x^2+2} dx$$

12. **[10 pts.]** Evaluate:

$$(a) \int_1^2 \left(x^3 - \frac{1}{x^2} + 3 \right) dx$$

$$(b) \int_1^e \left(\frac{2}{x} - 5 \right) dx$$

13. **[15 pts.]** Find the area of the region bounded by the curves $y = x^2 + 2x + 3$ and $y = 2x + 4$.