

Final Exam

Math 211

SPRING 2010

Name (print): _____

Signature: _____

Z-number: _____

Section: _____

Show all work for credit!

| | | |
|--------------|------|--------------------|
| Page 1 & 2 | / 25 | TOTAL / 200 |
| Page 3 | / 15 | |
| Page 4 | / 15 | |
| Page 5 | / 20 | |
| Page 6 | / 15 | |
| Page 7 | / 20 | |
| Page 8 & 9 | / 20 | |
| Page 10 | / 20 | |
| Page 11 | / 15 | |
| Page 12 & 13 | / 20 | |
| Page 14 | / 15 | |

1. **[25 pts.]** Find the derivative of each of the following functions.

(a) $f(x) = \left(\sqrt{x} + 2x^2 + \frac{5}{x} + e^x \right)$

(b) $f(x) = (e^x + 3x)^2 (x^2 - 2)^{10}$

(c) $f(x) = \left(\frac{x^2 + 5x}{5x - x^2} \right)^5$

(d) $f(x) = (x^3 - 3x)^2 \ln(2x^2 + 3)$

(e) $f(x) = \ln\left(\frac{x^3 + 3}{2x^2 - 3x}\right)$

2. **[15 pts.]** Let $f(x) = \begin{cases} -2x^2 + 1 & \text{if } x \leq 1 \\ x - 1 & \text{if } x > 1. \end{cases}$

(a) Evaluate $\lim_{x \rightarrow 1^-} f(x) =$

(b) Evaluate $\lim_{x \rightarrow 1^+} f(x) =$

(c) Does $\lim_{x \rightarrow 1} f(x)$ exist? Explain.

3. **[15 pts.]** Find the following limits (if they exist):

(a) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

(b) $\lim_{x \rightarrow 0} (4x^5 - 3x^2 + 2)$

(c) $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x}{5x^2 + 2x + 3}$

4. **[20 pts.]** Using the limit definition of the derivative of a function, find $f'(x)$ where

$$f(x) = \frac{1}{x+1}$$

5. **[15 pts.]** Find the equation of the tangent line to the graph of $f(x) = \left(\frac{2x+1}{x+1}\right)^2$ at $x = 0$.

6. **[20 pts.]** The quarterly profit (in thousands of dollars) of Cunningham Reality is given by

$$P(x) = -\frac{1}{3}x^3 + 16x + 20 \quad , \quad (0 \leq x \leq 50).$$

where x (in thousands of dollars) is the amount of money Cunningham spends on advertising per quarter.

(a) Find $P'(x)$.

(b) For which value of x , will the quarterly profit be at its maximum?

(c) What is the rate of change of quarterly profit if the amount Cunningham spends on advertising is $x = 10$ (thousands of dollars)?

7. **[20 pts.]** Let $f(x) = 2x^3 - 3x^2 - 12x - 8$.

(a) Determine $f'(x)$ and $f''(x)$.

(b) Find the relative maxima/minima of the function using the first derivative test.

(c) Use the second derivative test to determine where the graph of $f(x)$ is (i) concave upward and (ii) concave downward and (iii) find the value(s) of x for which an inflection point occurs.

(d) Draw a rough sketch of the graph using the information in (b) and (c).

8. **[20 pts.]** A carpenter is building a rectangular fence next to a river using 120 yd. of fencing (no fencing is needed along the river).

(a) What dimensions of the rectangle will maximize the area?

(b) What is the maximum area?

9. **[15 pts.]** The growth rate of some bacteria is proportional to its population size. Suppose 10,000 bacteria are present initially in the culture and 60,000 are present 2 hours later.

(a) Find the growth constant k .

(b) How many bacteria will there be after 4 hours?

(c) What is the rate of growth of the population at $t = 4$ hours. (Hint find $P'(4)$).

10. **[20 pts.]** Compute the following integrals:

$$(a) \int \left(x^2 - \frac{1}{2}\sqrt{x} + x^{-5} \right) dx$$

$$(b) \int (7e^{6x} - \sqrt{x}) dx$$

$$(c) \int_1^2 \left(x^2 - \frac{7}{x} + 2 \right) dx$$

$$(d) \int x^2 e^{x^3+3} dx$$

$$(e) \int \frac{(2x+2)}{x^2+2x+4} dx$$

11. **[15 pts.]** Find the area bounded by the curves:

$$f(x) = -x^2 + 2x + 3 \quad \text{and} \quad g(x) = -x + 3$$