Math 229, Spring 2011
Exam I Study Guide

Note: Section numbers have been changed to match the new edition of the text.

Exam I will cover sections 1.1-2.2 of the text, together with all assigned exercises and problems from those sections.

We haven't encountered much straight computation yet. Most of the exam will test your understanding of the basic notions of limit, continuity and derivative. A good deal of your study should involve *thinking* about the notions and the theorems we have concerning them, and explaining them to yourself or others until you truly *know* what they mean. If you reach this point, the exam should be easy for you.

Here are some indications of what we will expect from you on the exam, together with suggested problems from the text that will reinforce the relevant understanding and skills.

1. You will be required, as you have in the homework exercises, to produce (quickly and correctly) graphs of transformations of basic functions such as quadratics, power functions, absolute value, and the six trigonometric functions. You may need to graph piecewise-defined functions of which these constitute the pieces. You may need a fact about one of these functions that can be read off of its graph. (E.g. If you are asked for $\lim_{x\to\frac{\pi}{2}^+}\tan(x)$, we expect you to be able to answer $-\infty$ because you know the graph of the tangent function well.)

   On a related subject, we will expect you to know the values of the six trigonometric functions at the standard angles. (Thus, e.g., you should be able to compute $\lim_{x\to\pi/3}\cos(x) = \cos(\pi/3) = 1/2$.)

2. You should be able to determine, given (or having produced) the graph of a function, whether or not the function has a left-hand, right-hand or two-sided limit at a given point (including infinite limits). You should further be able to determine from a graph whether the function is continuous from the left, continuous from the right, or continuous at a given point.

3. You must know well the algebraic limit laws 1-11 on pages 62 to 64 of the text. Given that we will allow you to use the fact that algebraic functions are continuous on their domains (see below) to find limits of algebraic functions at points in their domains, we will be testing your understanding of these laws and theorems in problems where you do not have an algebraic formula for the function involved. Please see **1.6: 1, 2 and 56** for examples of how we might do this. These are excellent *reasoning* problems.

4. You will be allowed (and expected) to take as given that the trigonometric functions and the power functions are continuous on (i.e. at every point in) their domains, and that any function that is built from such functions by the application of the arithmetic operations, the taking of roots, and composition is again continuous on its domain. Thus, you may answer simply (and correctly) that

$$\lim_{x\to 0}\frac{(x+1)\cos(\sin(x)) - \sqrt{x^2+5}}{\tan(x)+3} = \frac{(0+1)\cos(\sin(0)) - \sqrt{0^2+5}}{\tan(0)+3} = \frac{1-\sqrt{5}}{3}$$

   because the argument of the limit is continuous on its domain, and 0 is in its domain.

   The problems, as you know, come in finding the limits of such functions at points that are *not* in their domains. You should be comfortable with the methods we have studied to date for finding such limits, if they exist, determining that they do not exist and KNOWING THE DIFFERENCE.

(a) What can you do with quotients whose numerator and denominator *both* have limit 0 at c? You should know to use the outlined principle on p.81 of the text to justify replacing the quotient with a function that agrees with the original away from c but whose numerator and denominator do not *both* go to 0.

(b) What if you have a quotient whose numerator has a nonzero limit at c, and whose denominator has limit 0^+ or 0^- (i.e. approaches 0 through positive or through negative values)? You should know that from each side, the function will have an infinite limit, and be able to determine, from each side, whether that infinite limit is ∞ or $-\infty$.

(c) What if you have a difference $f(x) - g(x)$, where both $f(x)$ and $g(x)$ have infinite limit ∞ at c? You should know that you must rewrite the argument $f(x) - g(x)$ somehow as something other than a difference, and reexamine the problem.

Practice problems: 1.6: 11-32, Ch.1 Review: 3-16

(d) Do you know how to find limits of piecewise defined functions by using the one-sided limits. (See, for instance, Example 7 on p. 55.) What if your limit has an argument involving an absolute value? Do you know how to deal with it by expressing the function as a piecewise defined function? (Good practice problems are **1.6: 41-46, 47-50 and 1.8: 17-22, 41-43, without appealing to graphs**.)

(e) Do you know the statement of the Squeeze Theorem and how to use it? (Practice problems are **1.6: 35-39**. What is it about these functions that forces you to use the Squeeze Theorem, rather than an arithmetic theorem?)

5. Can you state the Intermediate Value Theorem correctly, and do you *understand what it says*? You may be asked to use it to prove the existence of a root of an equation, or a zero of a function, in an interval. Study the presentation of Example 9 of Section 1.8, and solidify your understanding by working through several of problems **1.8: 45-54**.

6. You should know the definition of the average rate of change of a function f over an interval $[a, b]$, namely, $\frac{f(b)-f(a)}{b-a}$ and be able to identify this as the slope of the secant through the points $(a, f(a))$ and $(b, f(b))$. You should recognize that for $h \neq 0$, the function $\frac{f(a+h)-f(a)}{h}$ represents the average rate of change of f over the closed interval with endpoints a and $a+h$. (This would be $[a, a+h]$ if $h > 0$ and $[a+h, a]$ if $h < 0$.) Given an interval $[a, b]$ or a and h, you should be able to compute such an average rate of change.

   You should know the definition of the derivative $f'(a)$ of a function f at a point a as $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$, i.e. as the limiting value of the average rates of change $\frac{f(a+h)-f(a)}{h}$ as $h \to 0$, i.e. as the limiting value of the slopes $\frac{f(x)-f(a)}{x-a}$ of the secants through $(a, f(a))$ and $(x, f(x))$ as $x \to a$.

   You should be able, given a function f and a number a, to compute $f'(a)$ using this definition or to show that f is not differentiable at a by showing that the required limit does not exist.

   You should know that if $f'(a)$ exists, then the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ is DEFINED to be the line passing through $(a, f(a))$ with slope $f'(a)$. That is, BY DEFINITION, the slope of the tangent line to $y = f(x)$ at $(a, f(a))$ and the derivative $f'(a)$ of f at a are THE SAME THING.

You should be able, given a function f and a number a, to find an equation for the line tangent to $y = f(x)$ at the point $(a, f(a))$ by first computing the limit $f'(a)$ (the slope of the required line), and then producing the equation from the point and the slope.

If you can do the assigned homework exercises from Section 2.1 without looking anything up, you're in great shape here.

7. From Section 2.2, you will be responsible only for being able to produce the formulas for derivative functions $f'(x)$ as in exercises 19-29 on page 124. We will delay testing on the graphical interpretation of this function until you have had more experience.