Math 229 Final Examination Dec 9, 2011 Name: ________________

No books, notes, or calculators are allowed on this test. Your instructor will provide you with scratch paper, if you need it. Be sure that all of your work is shown and that it is well organized and legible.

(1) (12 pts) True or False. Here $f(x)$ and $g(x)$ denote functions from $\mathbb{R}$ to $\mathbb{R}$.

(a) If $f(x)$ is continuous at a, then $f(x)$ is defined at a.

(b) If $f(x)$ is differentiable at a, then $f(x)$ is continuous at a.

(c) If $f(x)$ is continuous at a, then $f(x)$ is differentiable at a.

(d) If $f(x)$ is continuous at a, then we cannot have $\lim_{x\to a} f(x) = \infty$.

(e) The square root curve $f(x) = \sqrt{x}$ is differentiable at $x = 0$.

(f) If $f'(x) = g(x)$, then $\int_a^b f(x)\,dx = g(b) - g(a)$.

(2) (20 pts) Compute the following limits or argue that they do not exist. If the limit is infinite, write ∞ or $-\infty$.

(a) $\lim_{x\to 3^+} \left(\frac{-5}{(x-3)^2} + \frac{1}{x-3} \right)$

(b) $\lim_{x\to 3^-} \frac{|2x-6|}{x-3}$

(c) $\lim_{x\to \pi^+} \frac{\sin x}{1+\cos x}$

(d) $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x}$

(3) (15 pts) Let $f(x) = \sqrt{2x-1}$.

(a) Use the limit definition of derivative to write $f'(5)$ as a limit.

(b) Compute the limit in part (a). [No credit for any other method.]

(4) (10 pts) Given a function $f(x) = \begin{cases} x^2 + 5c & \text{if } x < 3 \\ cx + 1 & \text{if } x \geq 3 \end{cases}$.

(a) Find $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$.

(b) For what value(s) of c is $f(x)$ continuous for all x?

(5) (16 pts) Suppose the functions $f(x)$ and $g(x)$ and their derivatives have the following values at $x = 0$ and $x = 1$:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	6	1	-2	1/2
1	4	5	3	2

Compute the derivatives of each of the following functions at the given value of x:

(a) For $h(x) = 3f(x) - 4g(x)$, find $h'(1)$

(b) For $h(x) = \dfrac{f(x)}{g(x)+1}$, find $h'(1)$

(c) For $h(x) = f(x + g(x))$, find $h'(0)$

(d) For $h(x) = f(x)\,(g(x))^2$, find $h'(1)$.

(6) (8 pts) Find the equation of the tangent line to $y = \tan\left(\frac{\pi}{4}x\right)$ at the point (1,1).

(7) (15 pts) Find the derivative of the following functions:

(a) $f(x) = \sin\big(\cos(\tan x)\big)$

(b) $f(x) = \dfrac{3x \sin x}{x^2 + 4}$

(c) $f(x) = (7x+1)^{-11} + \sqrt[3]{x^2} - \dfrac{1}{\pi^2}$

(8) (5 pts) Use implicit differentiation to find $\dfrac{dy}{dx}$ if $y \sin x = x \sin y$.

(9) (6 pts) Use linear approximation to estimate $\sqrt{82}$.

(10) (8 pts) Let $f(x)$ be a function with $f'(x) = x^3(x-1)^2(x-2)$.

(a) Find the interval(s) where $f(x)$ is increasing.

(b) Find the critical points of $f(x)$. Identify each critical point as a maximum, minimum, or neither.

(11) (8 pts) Let $g(x)$ be a function with $g''(x) = x^5(x-1)^{\frac{1}{3}}(x-2)^4$.

(a) Find the interval(s) where $g(x)$ is concave up.

(b) Find all inflection points of $g(x)$.

(12) (8 pts) Consider the function $f(x) = \dfrac{6x^3 + 7x - 8}{x^3 - 2x^2 - 3x}$.

(a) Find all horizontal asymptotes of $f(x)$ by computing $\lim_{x\to\infty} f(x)$.

(b) Find all vertical asymptotes of $f(x)$.

(13) (10 pts) A continuous function $f(x)$ on the interval $[0, 2]$ has the values:

$$f(0) = f(2) = 3, \quad f(\tfrac{1}{3}) = f(\tfrac{5}{3}) = \tfrac{5}{4}, \quad f(\tfrac{1}{2}) = f(\tfrac{3}{2}) = 0, \quad f(1) = -1.$$

The signs of the first and second derivatives of $f(x)$ are given in the following table:

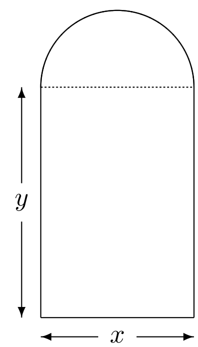
Interval	$(0, \frac{1}{3})$	$(\frac{1}{3}, \frac{1}{2})$	$(\frac{1}{2}, 1)$	$(1, \frac{3}{2})$	$(\frac{3}{2}, \frac{5}{3})$	$(\frac{5}{3}, 2)$
$f'(x)$	$-$	$-$	$-$	$+$	$+$	$+$
$f''(x)$	$-$	$+$	$+$	$+$	$+$	$-$

Sketch the graph of $y = f(x)$, carefully labeling all local maximums and minimums, absolute maximums and minimums, and inflection points.

(14) (10 pts) A plane is flying horizontally at an altitude of 1 mile and a speed of 500 mph directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when that distance is 3 miles.

(15) (6 pts) If $3 \leq f'(x) \leq 8$ for all x, use the Mean Value Theorem to find the biggest value and the smallest value that $f(4) - f(2)$ can possibly be.

(16) (12 pts) A Norman window has the shape of a rectangle surmounted by a semicircle. The diameter of the semicircle is equal to the width of the rectangle. (See the picture below.) The perimeter of the window is 30 ft.



(a) Write a formula expressing the total area of the window as a function of x and y.

(b) Write a formula that expresses the fact that the outer perimeter of the window is 30.

(c) Combine parts (a) and (b) to obtain a formula expressing the total area of the window as a function of the variable x only.

(d) For what value of x is the total area of the window a maximum?

(17) (6 pts) Approximate $\int_2^{10} (t-3)^2\, dt$ using the Riemann sum with $n = 4$ rectangles and right-hand endpoints.

(18) (25 pts) Compute the following integrals:

(a) $\int (1 + 3x + 4\sin x)\, dx$

(b) $\int_1^4 \frac{5x^3 + x}{x^3}\, dx$

(c) $\int \frac{x}{(5x^2+3)^4}\, dx$

(d) $\int \sec x\, (\sec x + \tan x)\, dx$

(e) $\int (\sin x)\, (\cos(\cos x))\, dx$