Math 229 Final Exam December 14, 2012 Name: ____________________

No graphing calculator $\geq$ TI89 nor cell phone calculator is allowed on this test. Be sure that all of your work is shown and that it is well organized and legible.

(1) (12 pts) True or False. Circle your answer.

(a) **T** **F** All continuous functions have derivatives.

(b) **T** **F** All continuous functions have antiderivatives.

(c) **T** **F** If $f(x)$ is differentiable for all x and $f(1) = f(-1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

(d) **T** **F** If a is a critical point of f, then f must have a maximum or a minimum at $x = a$.

(e) **T** **F** If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

(f) **T** **F** $\displaystyle\int_0^\pi \sec^2 x\,dx = \tan(\pi) - \tan(0)$.

(2) (24 pts) Find the following limits. If the limit is infinite, write ∞ or $-\infty$.

(a) $\displaystyle\lim_{x \to \pi^-} \csc(x)$

(b) $\displaystyle\lim_{x \to 0} \frac{\sin(2x)}{3x + 4x^2}$

(c) $\displaystyle\lim_{x \to \infty} \frac{\sqrt{4x^2 + 1}}{3x - 1}$

(d) $\displaystyle\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{5x + 1} - \sqrt{7 - x^2}}$

(3) (9 pts) Let

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 3x & \text{if } 0 \le x < 1 \\ 5 & \text{if } x = 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

(a) Find $\lim_{x \to 1} f(x)$ if it exists.

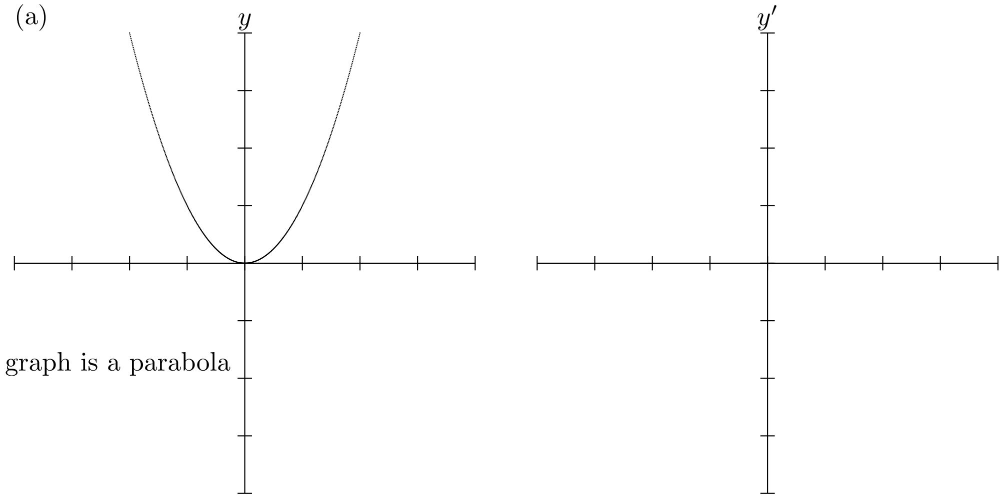
(b) Is $f(x)$ continuous at $x = 1$? Explain.

(c) Is $f(x)$ continuous at $x = 0$? Explain.

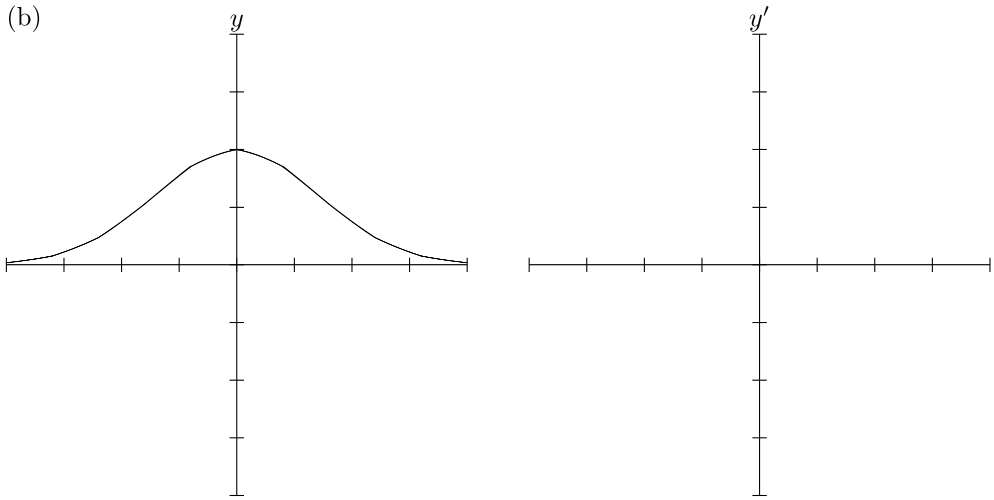
(4) (12 pts) Let $f(x) = \dfrac{x}{x+2}$. Use the limit definition of derivative (either version) to find $f'(3)$. [No credit for any other method.]

(5) (9 pts) In each part, the graph of a function is given. Draw a graph of the derivative of this function in the adjacent coordinate system.

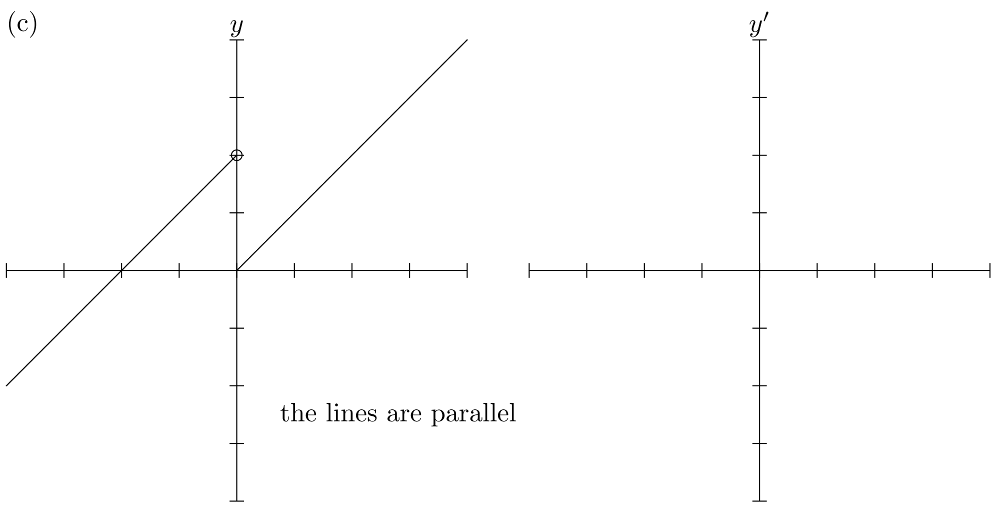
(a)



(b)



(c)



(6) (14 pts) Find the derivative of the following functions [You do not need to simplify]:

(a) $f(x) = \sqrt{\sin(x^4 + 16)}$

(b) $f(x) = \dfrac{x^7 + \frac{4}{x^2}}{2x - 7}$

(7) (8 pts) Find $\dfrac{d^2y}{dx^2}$ if $y = x \sec x$.

(8) (10 pts) Find the tangent line to the curve $xy^3 - 4xy = -6$ at the point (2,1).

(9) (10 pts) Find the critical points of the function $f(x) = x + 2\sin x$ on the interval $[0, 2\pi]$. Determine whether each critical point is a local maximum, minimum, or neither.

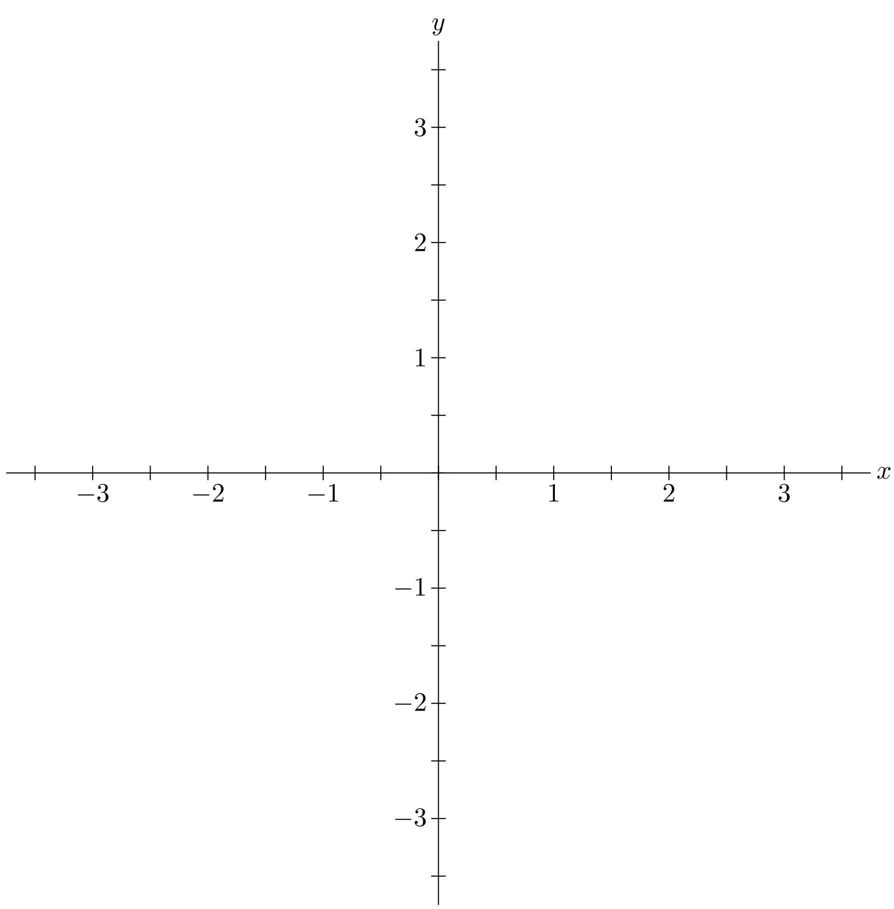
(10) (12 pts) The graph of the function $y = f(x)$ satisfies all of the following conditions:

- vertical asymptote $x = -1$
- horizontal asymptote $y = 1$
- $f(0) = 0$
- $f'(x) = \dfrac{2x}{(x+1)^3}$
- $f''(x) = \dfrac{2-4x}{(x+1)^4}$

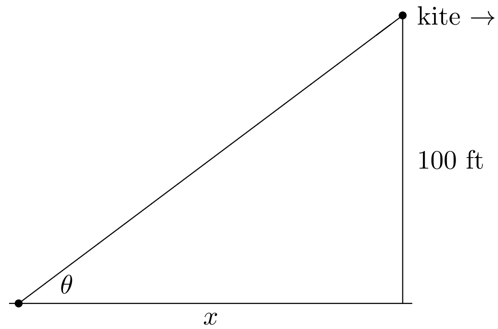
(a) Where is $f(x)$ increasing?

(b) Where is $f(x)$ concave up?

(c) Sketch the graph of $y = f(x)$. Display clearly the concavity of the curve, where it is rising and where it is falling. Label all asymptotes, local extrema, and inflection points.



(11) (12 pts) A kite 100 feet above the ground is flying horizontally at a speed of 8 ft/sec. At what rate is the angle θ between the string and the horizontal decreasing when 200 feet of string have been let out?

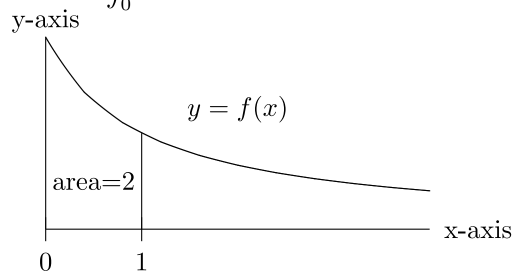


(12) (6 pts) Let $f(x) = x^5 - x - 1$. Use Newton's method with the initial approximation $x_1 = 0$ to find x_2 and x_3.

(13) (12 pts) Find two numbers x and y whose difference is 100 and whose product is a minimum.

(14) (6 pts) Approximate $\int_0^2 \frac{1}{1+x^2}\,dx$ using the Riemann sum with $n = 4$ rectangles and right-hand endpoints. Is the approximation an over-estimate or an under-estimate?

(15) (10 pts) For $x \geq 0$, define $g(x) = \int_0^x f(t)\,dt$, where f is the function graphed below.



(a) What is $g(0)$?

(b) What is $g(1)$?

(c) What is $g'(x)$?

(d) Is $g(x)$ decreasing or increasing?

(e) Is $g(x)$ concave up or concave down?

(16) (6 pts) Find the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ of the function $f(x, y) = \dfrac{xy}{x - y}$.

(17) (28 pts) Compute the following integrals:

(a) $\displaystyle\int_1^4 \frac{x^2-\sqrt{x}}{x}\,dx$

(b) $\displaystyle\int_0^{\pi/4} \frac{2\tan\theta - 3\sec^3\theta}{\sec\theta}\,d\theta$

(c) $\displaystyle\int \cos^4(3x)\sin(3x)\,dx$

(d) $\displaystyle\int t\sqrt{t^2+9}\,dt$