Math 229, FINAL EXAM, Spring 2011 **NAME**

1. (24 points) Compute $\frac{dy}{dx}$ in each of the following. You NEED NOT SIMPLIFY.

   (a) $y = x^2 \sec^3(5x)$

   (b) $y = \dfrac{\tan x}{x^3 + 4x^2 - 4}$

   (c) $y = \dfrac{5}{\sqrt[3]{x \sin x + 2}}$

   (d) $y = \cot(x^2) - \csc(3x) + \frac{1}{\sqrt{2}}$

2. (6 points) Given the curve $3x^2 + xy - y^3 = 9$, find the following:

   (a) a formula for $\frac{dy}{dx}$

   (b) an equation of the line tangent to the curve at the point $(1, -2)$.

3. (28 points) Compute the following limits. Show your work.

(a) $\lim_{x \to \infty} \frac{5\sqrt{x} + 4x^2}{11 + x^3}$

(b) $\lim_{x \to -\infty} \frac{\sqrt{10x^6 + 2}}{4x^3 + 7}$

(c) $\lim_{x \to 3} \frac{\sqrt{5x + 1} - 4}{x - 3}$

(d) $\lim_{x \to 0} \frac{\sin x}{x^3 + 5x}$

4. (28 points) Let $f(x) = \frac{1}{x^2} - \frac{3}{x} + 4$.

(a) Find any vertical and horizontal asymptotes of the graph of f.

(b) Find the intervals of increase and decrease of f and all points $(a, f(a))$ for which $f(a)$ is a local maximum or a local minimum.

(c) Find the intervals on which f is concave upward and those on which it is concave downward. Find all inflection points $(b, f(b))$ of f.

(d) Sketch a good graph of f that plots all intercepts, local extrema, inflection points and vertical and horizontal asymptotes, and is consistent with all your answers above.

5. (18 points) Find the following general antiderivatives.

(a) $\displaystyle\int \frac{\sin^2(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \frac{2}{\sqrt[5]{3x-4}}\,dx$

(c) $\displaystyle\int \frac{\csc^2(\frac{1}{x^3})}{x^4}\,dx$

6. (18 points) Evaluate the following definite integrals.

(a) $\int_1^4 \frac{x^2+1}{\sqrt{x}}\, dx$

(b) $\int_0^{\pi/4} \frac{\sec^2 x}{(\tan x + 2)^3}\, dx$

(c) $\int_1^5 \sin(\pi x)\, dx$

7. (8 points) Find the absolute maximum and absolute minimum values of the function defined by $f(x) = 9x + \frac{25}{x}$ on the interval $[-5, -1]$.

8. (8 points) Given that $f''(t) = \cos(t) + 1$, $\quad f'(0) = 3$, and $\quad f(0) = \sqrt{2}$, find $f(t)$.

9. (6 points) Let $g(x) = \displaystyle\int_{1/4}^{x} \tan(\pi\sqrt{t})dt$. Find $g'(1/9)$.

10. (8 points) Approximate the value of $\displaystyle\int_{0}^{\pi} \sin x \; dx$ using the Riemann sum with $n = 4$ and sample points taken to be right-hand endpoints.

11. (10 points) Use the **definition** of the derivative as a limit of average rates of change to compute $f'(x)$, where $f(x) = x^2 - 3x + 5$. (No credit will be given for a calculation using the power and sum rules.)

12. (8 points) Find a number c for which function f defined by $f(x) = \begin{cases} \sin(x) & \text{if } x \leq \pi/2 \\ 2x + c & \text{if } x > \pi/2 \end{cases}$ is continuous at $\pi/2$.

13. (8 points) Suppose that f is a differentiable function such that $f(1) = 7$ and $f'(x) < 2$ on the interval $(1, 10)$. Argue from an appropriate theorem that $f(10) < 25$.

14. (8 points) Find the radius (only) of the cylindrical closed soup can with a volume of 54 cubic inches whose total surface area (top plus bottom plus side) is as small as possible.

15. (8 points) Two cars are headed for the same intersection, one coming from the north at 65 miles per hour, and the other from the east at 100 miles per hour. At the time when the car coming from the north is 6 miles from the intersection, and the car coming from the east is 8 miles from the intersection, at what rate is the distance between the cars decreasing?

16. (6 points) Let $z = y^3 + 2x^2y + 6x - 5y$. Find $\frac{\partial z}{\partial y}$, the partial derivative of z with respect to y.