Math 229 Final Examination December 15, 2000

No books, notes, or calculators are allowed on this test.

Part I. Mechanics. In problems 1–15, your answer to each question will receive either 0, 4, or 5 points.

1. Find $\lim_{x\to\infty} \dfrac{\sqrt{9x^2-5}}{2x}$.

2. Find $\lim_{t\to 0} \dfrac{\sin^2 t}{t}$.

3. Find $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-3x+2}$.

4. If $y = 5\sec x$, find $\dfrac{dy}{dx}$.

5. If $g(t) = \dfrac{t^2}{t^2-1}$, find $g'(t)$.

6. If $z = \tan(\sqrt{\theta})$, find $\dfrac{dz}{d\theta}$.

7. If $f(x) = \sin 2x$, find $f''(x)$.

8. At *what value of* x does $f(x) = 2x^3 - 9x^2 + 12x$ have a *maximum* on the interval [0,3]?

9. What is the *minimum value of* $g(x) = (t-2)^{4/5} + 7$?

10. On what open interval(s) is the function $f(x) = -x^3 + 12x$ increasing?

11. On what open inteval(s) is the graph of the function $y = 2x^3 - 6x^2 + 1$ concave downward?

12. Integrate: $\displaystyle\int 21x^2(x^3+7)^5 dx$.

13. Integrate: $\displaystyle\int_0^{\pi/4} \sec^2 x\, dx$.

14. What is the area of the region above the x-axis and under the graph of $y = 6x^2 + 1$ over the interval $[-1, 2]$?

15. A particle moves in a straight line with acceleration $a(t) = 2t+3$ ft/sec^2. If its velocity at time $t = 0$ is 8 ft/sec, what is its velocity $v(t)$ at time t?

Part II. Partial credit will be given in grading problems 16 - 25. In order to receive as much credit as possible, be sure that all of your work is shown and that it is well organized and legible.

16. [12 pts] Use *the definition of derivative* to compute $f'(5)$ for the function $f(x) = x^2 - 3x$. [You will not receive any credit if you use a differentiation formula instead of the limit definition.]

17. [10 pts] Find an equation for the tangent line to the graph of $\sin(xy) = \frac{1}{2}$ at the point $(x, y) = (2, \frac{\pi}{12})$.

18. [15 pts] An open box is to made from a square piece of metal 12 inches on a side, by cutting equal squares from each corner and turning up the sides. Find the dimensions and volume of the largest box that can be made in this manner.

19. [15 pts] George W starts walking north at 4 ft/sec from a point P in Palm Beach, Florida. Thirty seconds later Al Gore starts walking south at 5 ft/sec from a point 500 ft due east of P. At what rate are George and Al moving apart 2 minutes after Al starts walking?

20. [8 pts] Find the linearization $L(x)$ of the function $f(x) = \sqrt{x}$ at $a = 25$ and use it to find an approximation for $\sqrt{26}$.

21. [8 pts] Differentiate: $y = (\sqrt{x^2+1})^3 \sin(x^3+1)$. [You don't need to simplify your answer.]

22. [8 pts] Integrate: $\displaystyle\int \frac{x}{\sqrt{x+4}}dx$

23. [16 pts] Sketch the graph of a function $f(x)$ that satisfies all of the following conditions:

$$\lim_{x\to\infty} f(x) = 0 \qquad \lim_{x\to-\infty} f(x) = 0 \qquad \lim_{x\to-1^+} f(x) = +\infty \qquad \lim_{x\to-1^-} f(x) = -\infty$$

$f(x) > 0$ on the intervals $(-\infty, -2)$ and $(-1, \infty)$

$f'(x) > 0$ on the intervals $(-\infty, -3)$ and $(0, 2)$

$f''(x) > 0$ on the intervals $(-\infty, -4)$, $(-1, 1)$, and $(3, \infty)$.

(For $x \neq -1$, it is assumed that $f(x)$, $f'(x)$, and $f''(x)$ are all ≤ 0 outside of the intervals specified above.) Clearly label all intercepts, asymptotes, max/min points, and inflection points on your graph.

24. [21 pts] (a) Explain why Newton's method *doesn't work* for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.

(b) Use the Fundamental Theorem of Calculus to compute $f(2)$ if $f(x)$ is continuous and satisfies $\displaystyle\int_0^x f(t)\,dt = x^2 + x$.

(c) Use the Mean Value Theorem to deduce that, regardless of the value of b, there *cannot be two* points in the interval $[-1, 1]$ for which

$$x^3 - 3x + b = 0.$$

25. [12 pts] Use the form for the integral

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$, to evaluate the integral $\displaystyle\int_0^5 (1+3x^2)dx$. [You will not receive any credit if you use an integration formula instead of the limit definition.]

Formulas: $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$