

1. (15 pts.) (a) State the limit definition of derivative.  
(b) Let  $f(x) = 3x - 2x^2 + 1$ . Find  $f'(x)$  using the limit definition.
2. (20 pts.) Find the derivatives of the following:  
(a)  $\sqrt{x^2 + 5}$     (b)  $\tan^2(3x - 7)$     (c)  $x^4 \sec(\sqrt{x})$     (d)  $\frac{\cos(2x)}{x}$
3. (15 pts.) Let  $f(x) = \begin{cases} \frac{x^2-9}{x-3} & x < 3 \\ 5 & x = 3 \\ 2x & x > 3 \end{cases}$ 
  - (a) Find  $\lim_{x \rightarrow 3^-} f(x)$ .
  - (b) Does  $\lim_{x \rightarrow 3} f(x)$  exist? Why or why not?
  - (c) Is  $f(x)$  continuous at  $x = 3$ ? Why or why not?
4. (10 pts.) Find the tangent line to the curve  $y + x^3 + y^3 = 38$  at the point  $(2, 3)$ .
5. (15 pts.) Find the maximum value of  $x^2y$  if  $x$  and  $y$  are positive and  $x + y = 10$ .
6. (10 pts.) A ball throw upward on planet X has height  $x = -5t^2 + 20t + 2$  meters after  $t$  seconds. How fast, and in which direction, is the ball moving at  $t = 3$  seconds?
7. (10 pts.) Use linear approximation or differentials to estimate  $\sqrt{25.04}$ .
8. (15 pts.) What is the area of the region between the curves  $y = x^2 + x$  and  $y = 2$ ?
9. (28 pts.) Let  $f(x) = \frac{1}{4 + x^2}$ .
  - (a) On what interval(s) is  $f(x)$  increasing?
  - (b) On what interval(s) is  $f(x)$  concave down?
  - (c) Does  $f(x)$  have any local extrema? If so, where?
  - (d) Does  $f(x)$  have any inflection points? If so, where?
10. (12 pts.) Use Riemann sums with  $n = 3$  rectangles and left-hand endpoints to estimate  $\int_0^{\pi/2} \cos x \, dx$ . Is this an overestimate or an underestimate?
11. (15 pts.) An inverted conical tank of water is 10 ft. tall and has an upper radius of 2 ft. Water is entering at the rate of 10 ft<sup>3</sup>/min. How fast is the water level changing when it is 5 ft. deep? The volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ .
12. (35 pts.) Calculate the following integrals
  - (a)  $\int_{-1}^1 4x^2 - 3x \, dx$
  - (b)  $\int \frac{x^2 + 3}{\sqrt{x}} \, dx$
  - (c)  $\int x \sec^2(x^2) \, dx$
  - (d)  $\int_0^1 \sin(\pi x) \, dx$
  - (e)  $\int_0^1 \frac{x^2}{(x^3 + 1)^2} \, dx$