

Please show all your work and display your answers clearly.

1. (15 pts.) Find the following limits. Justify your answers.

(a)  $\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x + 2} - 2}$

(b)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x \cos 3x}$

(c)  $\lim_{x \rightarrow 3^-} \frac{4x - x^2}{x - 3}$

2. (20 pts.) Compute the derivatives of the following functions. Do not simplify your answers.

(a)  $y = x^{1/3} - \frac{3}{x^4} + \pi^{-1}$

(b)  $y = \sqrt{1 + \frac{x^5}{\cos x}}$

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(c)  $y = (x + 1)^{2/3}(5x^4 - 1)$

(d)  $y = \int_x^{\pi/2} \left( \frac{\sin t}{4 - \cos t} \right)^2 dt$

3. (21 pts.) Find the following indefinite integrals and definite integrals.

(a)  $\int \frac{3x - x^2}{\sqrt[3]{x}} dx$

(b)  $\int_0^1 \frac{x^2}{\sqrt{x^3 + 2}} dx$

(c)  $\int_{\pi/4}^{\pi/2} \sin^3 x \cos x \, dx$

4. (12 pts.) Use the definition of derivative to find  $f'(3)$  for the function  $f(x) = x^2 - x$ . (No credit will be given for any other method.)

5. (10 pts.) Let  $f(x)$  be defined by  $f(x) = \begin{cases} 3x^3 - 2, & x \geq 1 \\ (x + 3)^2, & x < 1 \end{cases}$ .

(a) Is  $f(x)$  continuous at  $x = 1$ , and why?

(b) Is  $f(x)$  differentiable at  $x = 1$ , and why?

6. (6 pts.) Assume that the equation  $2xy - y^3 + 2 = 0$  defines a function  $y = f(x)$ . Find  $\frac{dy}{dx}$ .

7. (8 pts.) Find an equation for the tangent line to the curve  $y = x^4 - 7x + 2$  at  $x = 1$ .

8. (12 pts.) Find the maximal and minimal values of the function  $y = \frac{x}{x^2 + 1}$  on the interval  $[0, 3]$ .

9. (14 pts.) Sketch the graph of a function  $f(x)$  defined on  $(-2, \infty)$  satisfying:
- (i)  $f(-1) = f(1) = 0$ ,  $f(0) = -1$ ,  $f(2) = 2$ , and  $f(3) = 1$ ;
  - (ii)  $f'(x) < 0$  for  $x < 0$  and for  $x > 2$ ,  $f'(x) > 0$  for  $0 < x < 2$ ;
  - (iii)  $f''(x) > 0$  for  $x < 1$  and for  $x > 3$ ,  $f''(x) < 0$  for  $1 < x < 3$ .
  - (iv)  $\lim_{x \rightarrow \infty} f(x) = 0$  and  $\lim_{x \rightarrow -2^+} f(x) = \infty$ .

Clearly label the local extrema, inflection points, and asymptotes.

10. (8 pts.) A curve is defined by the function  $f(x) = \frac{x - 3x^2}{x^2 + x - 6}$ .

(a) Find the horizontal asymptote of this curve.

(b) Find the vertical asymptotes of this curve.

11. (12 pts.) A person 6 ft tall is walking away from a street light 20 ft high at the rate of 7 ft/sec. At what rate is the length of the person's shadow increasing?

12. (12 pts.) A box is to be constructed which has an open top and a rectangular base whose length is 4 times its width such that the volume of the box is  $3200 \text{ cm}^3$ . Find the dimensions of the box that minimize the amount of the material needed for its construction (i.e., material needed for its surface area).

13. (6 pts.) The graphs of  $f$ ,  $f'$ , and  $f''$  are given on the same coordinate plane. Indicate which is which and justify your answer.

14. (14 pts.) A particle is moving along a straight line at the velocity  $v(t) = t^2 + t - 2$ .

(a) Find the displacement from  $t = 0$  to  $t = 3$ .

(b) Find the total distance traveled from  $t = 0$  to  $t = 3$ .

15. (12 pts.) Approximate the the area under the curve  $y = \frac{1}{4-x}$  on the interval  $[1, 3]$  by using a Riemann sum with a partition of four subintervals of equal length and with right endpoints as the sample points. Is this estimate an underestimate or overestimate?

16. (12 pts.) Calculate the area of the region bounded by the curves  $y = 6 - x^2$  and  $y = x$ .

17. (6 pts.) Express the limit  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{n} \sin\left(i\frac{\pi}{n}\right)$  as a definite integral.