

MATH 229
Spring 2007
FINAL EXAM

Name _____

1. [9 pts.] Evaluate the following limits

(a) $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 + 5x + 6}$

(b) $\lim_{x \rightarrow +\infty} \frac{3x^2 + x - 4}{7x^2 + 2x + 1}$

(c) $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

2. [10 pts.] Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{3x + 2}$

3. [16 pts.] Find the derivatives of each of the following:

(a) $x^2 \tan(2x)$

(b) $\frac{\sin x}{x^2 + 1}$

(c) $\cos^2(x^3)$

(d) $x^2 - 4x + \frac{2}{\sqrt{x}}$

4. [10 pts.] Find the tangent line to $y = \frac{x^2}{1 + x^2}$ when $x = 1$.

5. [15 pts.] Let $f(x) =$

$$\begin{cases} 4 - 2x & \text{if } x \leq 1 \\ x^2 + 1 & \text{if } 1 < x < 3 \\ x + 7 & \text{if } x > 3 \end{cases}$$

(a) Does $\lim_{x \rightarrow 3} f(x)$ exist? If so, what is it?

(b) Does $\lim_{x \rightarrow 1} f(x)$ exist? If so, what is it?

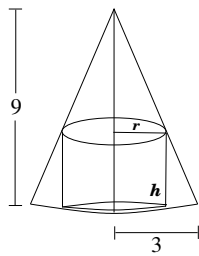
(c) Is $f(x)$ continuous at $x = 3$? Why or why not?

6. [10 pts.] Use implicit differentiation to find $\frac{dy}{dx}$ if $x + 2y + \sin(xy) = 5$.

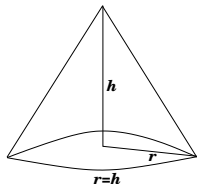
7. [10 pts.] Find the maximum and minimum values for $f(x) = 4x^3 - x^4 - 10$ on the interval $[1, 4]$.

8. [10 pts.] Use linear approximation (differentials) to estimate $\frac{1}{\sqrt{4.08}}$

9. [15 pts.] Find the maximum volume of a cylinder inscribed in a cone of radius 3 and height 9.



10. [15 pts.] Grain from a conveyor belt is falling onto a pile at the rate of $10 f^3/5$. The pile forms a cone with radius equal to the height. How fast is the height of the pile growing when it is $10f$ tall?



11. [10 pts.] Let $f(x) = 2x^3 - x^4 + 1$

(a) Find where $f(x)$ is increasing/decreasing

incr. _____

decr. _____

(b) Find where $f(x)$ is concave w/concave down.

up _____

down _____

12. [10 pts.] Using Newton's method with $x_1 = 2$ to approximate a root of $x^3 - 3x - 3 = 0$, find x_2 . Why is $x_1 = 1$ a bad choice?

13. [15 pts.] Draw a sketch of $y = f(x)$ if $f(x)$ has the following properties:

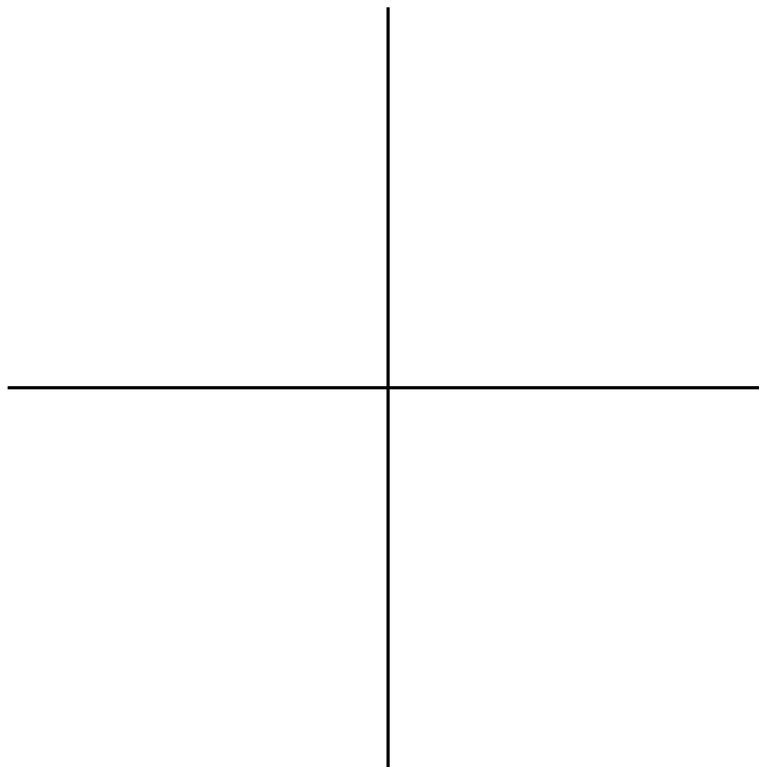
(a) $\lim_{x \rightarrow -\infty} f(x) = 0$, $\lim_{x \rightarrow +\infty} f(x) = -1$

(b) $f(x)$ has a vertical asymptote at $x = 1$

(c) $f(-2) = 2$, $f(0) = 4$

(d) $f'(x) > 0$ on $(-\infty, 0) \cup (1, +\infty)$; $f'(x) < 0$ on $(0, 1)$

(e) $f''(x) > 0$ on $(-\infty, -2)$; $f''(x) < 0$ on $(-2, 1) \cup (1, +\infty)$



14. [10 pts.] Suppose $f'(x) = \sin 2x + \sqrt{x}$ and $f(0) = 0$. Find $f(x)$.

15. [10 pts.] Find the area between the curves $y = 4x^2$ and $y = x^3$.

16. [10 pts.] Approximate $\int_1^3 \frac{1}{x} dx$ using $n = 4$ rectangles and right hand endpoints. (Riemann sum).

17. [15 pts.] Evaluate the following:

(a) $\int \frac{2x - 4}{\sqrt{x}} dx$

(b) $\int_0^{\pi/4} \sec^2 x dx$

(c) $\int_0^4 x\sqrt{x^2 + 9} dx$