

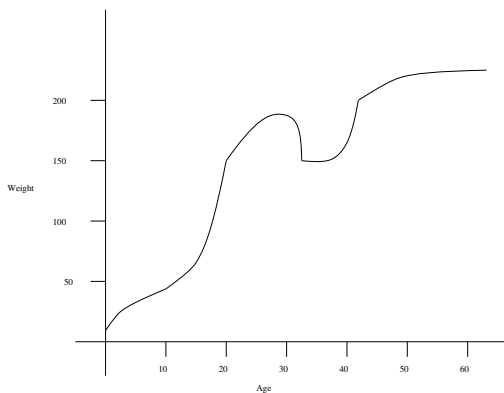
MATH 229 SUPPLEMENTARY EXERCISES

NIU MATHEMATICAL SCIENCES DEPARTMENT

These are suggested exercises which are not from the text. They are given roughly in the order covered in the course. These should not be viewed as “extra credit” type problems; they cover important course objectives and content not dealt with adequately in the text’s exercise sets.

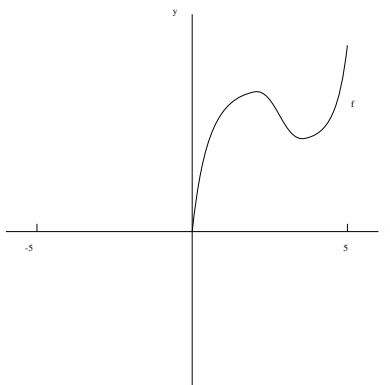
1. PROBLEMS DEALING WITH MATERIAL FROM CHAPTER 1

- Write an equation for each line described.
 - The line passing through $(1,2)$ with slope $-1/3$.
 - The line passing through $(1,2)$ and $(-1,3)$.
 - The line passing through $(1,2)$ that is parallel to the line given by $2x - 3y = 1$.
- Is there anything special about the relationship between the lines given by $Ax + By = C_1$ and $Bx - Ay = C_2$? (Here A and B are non-zero numbers.)
- How do you find an equation for a line given two points on the line? What if you’re given one point and the line’s slope? Give examples.
- How are the slopes of mutually perpendicular lines related?
- The graph below gives the weight of a certain person as a function of age. Describe in words how this person’s weight varies over time. What might have happened when this person was 30 years old?

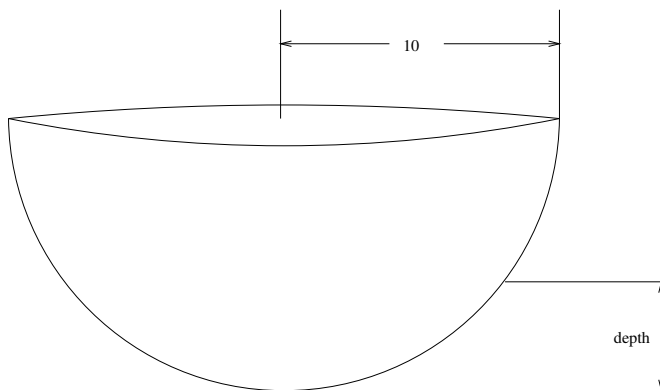


- What is an even function? What is an odd function? What symmetries do the graphs of such functions have? Give an example of a function which is neither even nor odd.

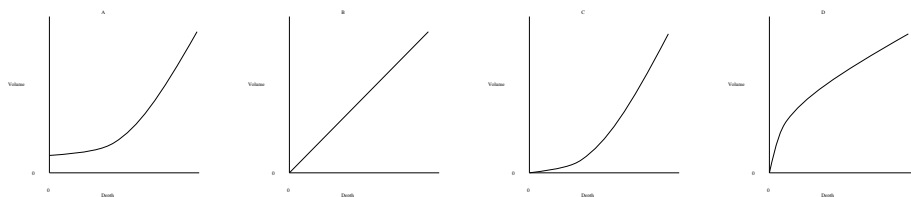
Date: Fall 2009.



7. A function f has domain $[-5, 5]$ and part of its graph is given above. Complete the graph given that f is: a) even, and b) odd.

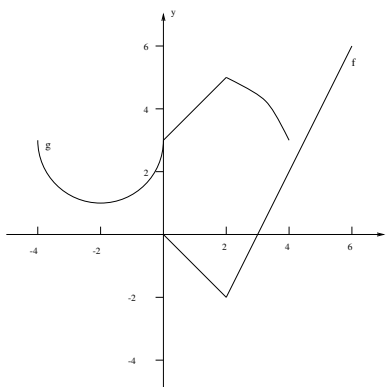


8. Let d denote the depth of water and V be the volume of water in the hemispherical tank above. Regard V as a function of d . Which graph below could be the graph of $V(d)$ (assume each graph uses the same linear scales)? Explain.



9. Sketch the graph of body temperature as a function of time which is consistent with the following scenario: “the child’s temperature has been rising for the last two hours, but not as rapidly since we gave her an antibiotic an hour ago.”

10. When is it possible to compose one function with another? Does the order in which they are composed matter?



11. Use the graphs of f and g above to evaluate each of the following compositions (or explain why it is undefined).

- | | |
|--------------|--------------|
| a) $f(g(2))$ | d) $g(f(6))$ |
| b) $f(g(0))$ | e) $f(f(4))$ |
| c) $g(f(0))$ | |

12. How do you change the equation $y = f(x)$ to shift its graph up or down? Give examples.

13. How do you change the equation $y = f(x)$ to shift its graph to the left or right? Give examples.

14. Find a viewing window such that the graph of $y = \sin x$ on your calculator looks like a straight line. Is this a good graph? If so, say why. If not, what makes it not a good graph?

15. You expect that the values of the functions $f(x)$ and $g(x)$ differ by a constant on some interval $a \leq x \leq b$. How could you use your calculator to support or refute this?

16. Graph $y = \sin x$ and $y = \sqrt{\sin^2 x}$ together for $-\pi \leq x \leq \pi$. On which intervals are these graphs the same? On which are they different? When they are different, is there any relationship you see between them? Explain.

17. If f is an odd function, can you conclude anything about the value of f at 0 (assume 0 is in the domain of f)? Explain.

2. PROBLEMS DEALING WITH MATERIAL FROM CHAPTER 2 AND THE BEGINNING OF CHAPTER 3

18. What is the difference between instantaneous and average velocity?

19. Explain in your own words what it means to say that $\lim_{x \rightarrow 2} f(x) = 5$.

20. Sketch the graph of a function which satisfies all of the following conditions:

$$\lim_{x \rightarrow 3^+} f(x) = 4, \quad \lim_{x \rightarrow 3^-} f(x) = 2, \quad \lim_{x \rightarrow -2} f(x) = 2,$$

$$f(3) = 3, \quad f(-2) = 1.$$

21. Sketch the graph of

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \sin(1/x) & \text{if } x > 0. \end{cases}$$

(You may want to use your calculator, but be somewhat careful on this one.) Does $\lim_{x \rightarrow 0^+} f(x)$ exist? If so, what is it? If not, why? Does $\lim_{x \rightarrow 0^-} f(x)$ exist? If so, what is it? If not, why? Does $\lim_{x \rightarrow 0} f(x)$ exist? If so, what is it? If not, why?

22. Do the same as in #21 with

$$f(x) = \begin{cases} 0 & \text{if } x = 0, \\ x \sin(1/x) & \text{if } x \neq 0. \end{cases}$$

23. If $\lim_{x \rightarrow 1} f(x) = 5$, must f be defined at 1? If it is, must $f(1) = 5$? In fact, can you conclude anything about the value of f at 1? Explain.

24. If $f(1) = 5$, must $\lim_{x \rightarrow 1} f(x)$ exist? If it does, must it be 5? In fact, can you conclude anything about the limit of $f(x)$ as x approaches 1? Explain.

25. Suppose f and g both have domain all the real numbers and that $\lim_{x \rightarrow 0} f(x)/g(x) = 1$. Is it possible for $f(0) = 0$? Can $g(0) = 0$? Do the values of f and g at 0 matter at all?

26. What can you say about the limit of a polynomial? A rational function?

27. What do $\lim_{x \rightarrow c} f(x) = \infty$ and $\lim_{x \rightarrow c} f(x) = -\infty$ mean?

28. Why bother with all that ε and δ stuff and the “formal” definition of a limit, anyway?

29. For each of the following functions, explain why it is continuous or discontinuous:

- the temperature at a specific location as a function of time;
- the temperature at a specific time as a function of the distance due west from Chicago;
- the taxi fare as a function of distance travelled.

30. Give an example of a function $f(x)$ that is continuous everywhere except at $x = 2$, where it has a removable discontinuity. Explain why f is discontinuous at 2 and why the discontinuity is removable.

31. If f and g are both continuous on $[0, 1]$, could $f(x)/g(x)$ possibly be discontinuous anywhere on $[0, 1]$? If not, why? If so, how?

32. Is it true that a continuous function that is never 0 is either always positive or always negative? Give reasons for your answer.

33. How do you test a function for continuity at a point?

34. What does it mean for a discontinuity to be “removable?”

35. Do the following two statements mean the exact same thing?

Not all functions are differentiable.

All functions are not differentiable.

What about these two statements?

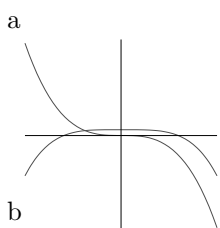
The function f is not continuous at any point in its domain.

The function f is discontinuous at every point in its domain.

36. How do we define slopes and tangent lines? What makes these definitions reasonable?

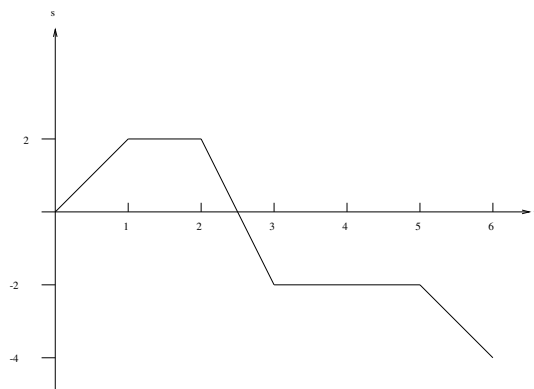
37. In your experience so far, when does a function typically not have a derivative at a point? What does this look like graphically?

38. Below are the graphs of a function f and its derivative f' . Which is which? Give reasons for your answer.



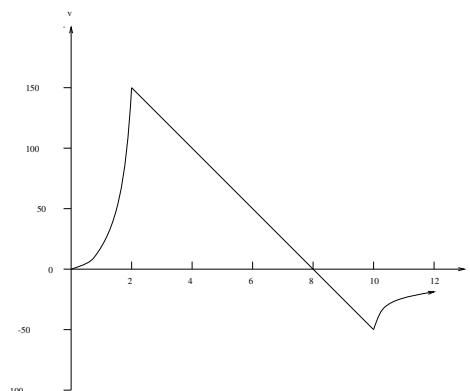
3. PROBLEMS DEALING WITH MATERIAL FROM CHAPTERS 3 AND 4

39. What rules and formulas have you learned for taking derivatives? Is it a long list? Can you shorten your list any and still be able to compute the same derivatives?



40. The graph above is for the position s of a particle as it moves along a line (as a function of time t). When is the particle moving to the right? To the left? When is it standing still? Graph the particle's velocity and speed as functions of time.

41. Find $\frac{d^{29} \cos(x)}{dx^{29}}$. Do you really need to take the first 28 derivatives here?



42. The graph above is for the velocity v (in feet/second) of a model rocket as a function of time t (in seconds) after launching. Use it to answer the following questions.

- At what point does the rocket's fuel run out?
- How fast is the rocket going when its fuel runs out?
- When did the rocket reach its highest point?
- When did the rocket's parachute deploy? How long had the rocket fallen when the parachute deployed?
- When was the rocket's acceleration greatest? When was it least?

43. Once you know the derivative of the sine and cosine function, explain how you can determine the derivatives of the remaining four trig functions. What if you only know the derivative of the sine function?

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$1/3$
1	3	-4	$-1/3$	$-8/3$

44. The chart above gives the values of f , g and their first derivatives at 0 and 1. Use it to find, if possible, the derivatives of the following functions at the given point.

- $f(x)g^3(x)$ at $x = 0$.
- $f(g(x))$ at $x = 0$.
- $f(x + g(x))$ at $x = 0$.
- $(x^{11} + f(x))^{-2}$ at $x = 1$.

45. Suppose that $u = g(x)$ is differentiable at $x = 1$ and $y = f(u)$ is differentiable at $u = g(1)$. Suppose further that the graph of $y = f(g(x))$ has a horizontal tangent at $x = 1$. What, if anything, can you conclude about the graphs of $u = g(x)$ and $y = f(u)$?

46. What is the chain rule? When does it apply? How do you use it?

47. What is implicit differentiation and when is it called for?

48. What is linearization and why do we bother with it?

49. How do you use differentials to estimate absolute change, relative change, and percent change?

50. Suppose you're using Newton's method to estimate roots of $f(x) = 0$ and you get really lucky: your first "guess" x_0 really is a root! What happens to x_1 and later approximations?

51. For fun you are going to use Newton's method to show your parents a good approximation to $\pi/2$ (by solving $\cos x = 0$). Does it matter what your starting value x_0 is? Give reasons for your answer.

52. What is Newton's method? What is the theory behind it? What are some of its strengths and limitations?

53. Is it true that a tangent to a curve can only touch the curve in one point? Can the tangent cross the curve at the point of tangency? Explain. Use examples as needed to illustrate your explanation.

54. Graph $f(x) = -2\cos x - \cos^2 x$ together with its derivative. Comment on the behavior of the graph of f in relation to the signs and values of f' .

55. We know how to find extreme values of a continuous function on a closed interval by looking for critical points and boundary points. But what if f is defined on an open interval (so there are no boundary points) and has no critical points? Is this possible? If so, what happens? Can you give examples of such functions?

56. Suppose f is differentiable on $[a, b]$ and $f(a) < f(b)$. Can you say anything about f' on $[a, b]$?

57. Suppose $f'(x) = 2x$ for all x . Find $f(2)$ if:

- a) $f(0) = 0$,
- b) $f(1) = 0$,
- c) $f(-2) = 3$.

58. How does the Mean Value Theorem relate average to instantaneous change?

59. What does the Mean Value Theorem tell us about functions with positive derivatives? Negative derivatives? Derivatives identically zero? Functions with identical derivatives?

60. How do you locate the vertical asymptotes of a rational function?

4. PROBLEMS DEALING WITH MATERIAL FROM CHAPTER 5

61. What is a general antiderivative?

62. How does knowing one of a function's antiderivatives help you find others?

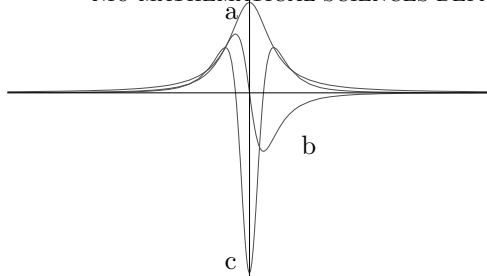
63. What is a Riemann sum? Why might you consider such a sum?

64. What are the rules for integrating sums, differences and constant multiples of integrable functions?

65. How do we define the area of the region between the graph of a continuous function $f(x)$, $a \leq x \leq b$, and the x -axis? Why define it that way?

66. Sketch the graph of a continuous function f which satisfies

$$\int_{-2}^3 f(x) dx = -10 \quad \text{and} \quad \int_0^3 f(x) dx = 0.$$



67. Above are the graphs of a function f , its first derivative f' and one of its antiderivatives F . Which is which?

68. Suppose that $f'(x)$ is negative for all values of x and that $f(1) = 0$. Let

$$h(x) = \int_0^x f(t) dt.$$

Which of the following statements must be true? Give reasons for your answers.

- h is a twice-differentiable function of x .
- Both h and its first derivative h' are continuous.
- The graph of $y = h(x)$ has a horizontal tangent at $x = 1$.
- h has a local maximum at $x = 1$.
- h has a local minimum at $x = 1$.
- The graph of $y = h(x)$ has a point of inflection at $x = 1$.
- The graph of $y = h'(x)$ crosses the x -axis at $x = 1$.

69. What is the Fundamental Theorem of Calculus and why is it important?

70. Let

$$f(x) = \int_0^x \frac{1}{\sqrt{1-t^2}} dt$$

for $-1 < x < 1$.

- What is $f'(x)$? What happens to the graph of $y = f(x)$ as x approaches 1? As x approaches -1 ?
- Use your calculator's numerical integration routine to compute some values of $f(x)$ for x near 1 (but less than 1, of course). Can you make any predictions about $\lim_{x \rightarrow 1^-} f(x)$?
- Suppose for the time being that $f(1)$ makes sense. Use the substitution $t = \sin \theta$ (so $dt = \cos \theta d\theta$ - what are the new limits of integration?) and see what you get. How does that compare to your computations and predictions above?
- What is $f(1/2)$? (Use the same substitution.)

71. Evaluate $\int \sec^2 x \tan x dx$ two ways: using the substitution $u = \tan x$ and the substitution $u = \sec x$. Are your answers the same? Did something go wrong here? Explain.

72. How does the substitution method for finding integrals work? How is it related to the chain rule?

73. Is it true that every function $f(x)$ that is differentiable on an open interval (a, b) is itself the derivative of some function on (a, b) ?