

Math 230 Spring 2010

Final Exam

Name: _____

Directions: Please clearly print your name above. There are 8 pages, including this cover sheet. There are 11 problems (some with multiple parts) for a total of 200 points; the point value for each individual problem is clearly marked. You must show your work to receive full credit. Scratch paper is not allowed. You may use the back of the page if you require more space to work a problem.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

total: _____

1 (16 points). Set up, but do not evaluate, an integral for computing the volume of the solid obtained by rotating the region bounded by $y = e^x$, $x = 0$ and $y = 2$ about the line $x = 1$. State which method you use to get your integral, and sketch a typical washer or shell, as appropriate.

2 (20 points). Find dy/dx for the following functions, showing all steps.

a) $y = e^{x^2} \sin^{-1} x$

b) $y = \frac{\tan^{-1} x}{\ln(1+x)}$

3 (14 points). Show that the function $f(x) = x^5 + x - 4$ is one-to-one and find $(f^{-1})'(-2)$.

4 (18 points). Evaluate the following limits.

a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{e^x - 1}$

b) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x$

5 (32 points). Evaluate the following integrals.

a) $\int_2^3 \frac{1}{x^2 - 1} dx$

b) $\int_0^\pi e^x \sin x dx$

c) $\int_e^\infty \frac{1}{x(\ln x)^2} dx$

d) $\int \frac{t^3}{\sqrt{t^2 + 1}} dt$

6 (24 points). Determine whether the following series converge absolutely, converge conditionally, or diverge.

a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3 + 2}$$

b)
$$\sum_{n=1}^{\infty} \frac{n^4}{e^n}$$

c)
$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n} - 1}$$

7 (16 points). Compute the second degree Taylor polynomial $T_2(x)$ for $f(x) = \tan^{-1}(x^2)$ centered at 0.

8 (12 points). Suppose $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 4$ and diverges when $x = -6$. What, if anything, can you say about the following series? Be sure to state your reasoning.

a) $\sum_{n=0}^{\infty} c_n 5^n$

b) $\sum_{n=0}^{\infty} c_n (-1)^n 9^n$

c) $\sum_{n=0}^{\infty} n c_n 3^{n-1}$

9 (20 points). Use power series to approximate $\int_0^{1/10} e^{-x^2} dx$ with an error less than 10^{-5} . Explain why your estimate is good enough. (Feel free to express your estimate as a sum of fractions.)

10 (14 points). Write out the first 4 terms of the sequence $a_n = 10^n/n!$ and find $\lim_{n \rightarrow \infty} a_n$.

11 (14 points). Find the radius of convergence of $\sum_{n=0}^{\infty} (-1)^n \frac{n}{4^n} (x+1)^n$.