MATH 230 FINAL EXAM (Sping 2011)

NAME: **ZID:** **Sec.**

20 questions, 10 points for each question. Show all work for full credit.

1. Find the volume of the solid generated when the region bounded by $y = e^x$, $y = e$, and $x = 0$ is rotated about the x-axis. Use washer method.

2. Evaluate $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$.

3. Find the derivative of $y = \tan^{-1}(e^x) + e^{\sin^{-1} x}$.

4. Find the derivative of $y = (1 + x^2)^x$.

5. The half-life of Radium-226 is 1590 years. Suppose you have a 100-mg sample. When will the mass be reduced to 15-mg?

6. Evaluate $\int \cos^4 x \sin^3 x dx$.

7. Evaluate $\int x \ln x dx$.

8. Evaluate $\int \frac{1}{x^2\sqrt{9-x^2}} dx$.

9. Evaluate $\int \frac{x-9}{(x+5)(x-2)} dx$.

10. Write the definition and evaluate the improper integral $\int_1^\infty xe^{-x^2}\,dx$ if it is convergent.

11. Find the length of $y = \ln(\cos x)$, $0 \le x \le \pi/3$.

12. Given $f(x) = 2x^3 + 5$, show that f is one-to-one and find f^{-1}.

13. Determine whether the series $\Sigma_{n=1}^{\infty} \frac{3^n}{n!}$ is convergent or divergent (mention the theorem you use).

14. Determine whether the series $\Sigma_{n=1}^{\infty}(1 + \frac{1}{n})^n$ is convergent or divergent (mention the theorem you use).

15. Show that the series $\Sigma_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^{n+1}}$ is a convergent geometric series and find the sum.

16. Determine whether the series $\Sigma_{n=1}^{\infty}(-1)^n \frac{n}{n^2+1}$ is absolutely convergent, conditionally convergent, or divergent (mention the theorems you use).

17. Determine whether the series $\Sigma_{n=1}^{\infty} \ln \frac{n}{n+1}$ is convergent by definition.

18. Find the interval of convergence for $\Sigma_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt{n}}$.

19. Find the 3rd-degree Taylor polynomial of $\sqrt{2x+1}$ at $a = 0$.

20. Find the Maclaurin series for $\sin x^2$ and use it to evaluate $\int \sin x^2 dx$.