

Math 230 Spring 2012

Final Exam

Name: _____

Directions: Please clearly print your name above. There are 7 pages, including this cover sheet. There are 12 problems (some with multiple parts) for a total of 200 points; the point value for each individual problem is clearly marked. You must show your work to receive full credit. Scratch paper is not allowed. You may use the back of the page if you require more space to work a problem.

1. _____

2. _____

3. _____

4. _____

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8. _____

9. _____

10. _____

11. _____

12. _____

total: _____

1 (16 points). Set up, but do not evaluate, an integral for computing the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = x$, $x = 4$, and $x = 9$ about the line $y = -2$. State which method you use to get your integral, and sketch a typical washer or shell, as appropriate.

2 (18 points). Find dy/dx for the following functions, showing all steps.

a) $y = 2^x \ln(1 + x^2)$

b) $y = \frac{\sin^{-1} x}{\tan^{-1} x + e^x}$

3 (14 points). Show that the function $f(x) = x^3 + x$ is one-to-one, then find the slope of the tangent line to the graph of $y = f^{-1}(x)$ at the point where $x = 2$.

4 (20 points). Evaluate the following limits.

a) $\lim_{x \rightarrow \infty} \frac{x^4}{e^x - 1}$

b) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{-2x}$

5 (24 points). Evaluate the following integrals.

a) $\int_0^1 x^3 \sqrt{1-x^2} dx$

b) $\int x^2 e^x dx$

6 (12 points). Determine whether the improper integral $\int_0^\infty \frac{e^{-x}}{x+1} dx$ converges or diverges. State your reasoning.

7 (16 points). Find a polynomial that approximates e^{-x} to within 10^{-4} for all x in the interval $[0, 1]$. Explain why your polynomial has the required accuracy.

8 (16 points). Suppose $\sum_{n=0}^{\infty} c_n(x-1)^n$ converges when $x = -4$ and diverges when $x = 8$. What, if anything, can you say about the following series? Be sure to state your reasoning.

a) $\sum_{n=0}^{\infty} c_n$

b) $\sum_{n=0}^{\infty} c_n(-1)^n 9^n$

c) $\sum_{n=0}^{\infty} c_n(-1)^n 6^n$

d) $\sum_{n=0}^{\infty} n c_n 3^{n-1}$

9 (10 points). What does it mean to say a series converges absolutely?

10 (20 points). Determine whether the following series converge absolutely, converge conditionally, or diverge. Be sure to state your reasoning.

a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2n^3 + 2}$$

b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n 100^n}{n!}$$

11 (24 points). Find power series for the following functions, and state the radii of convergence.

a) $\ln(1 + x^2)$

b) $\frac{1 - \cos x}{x^2}$

c) $\frac{1}{(1 - x)^2}$

12 (10 points). The graph $y = f(x)$ of a function is shown. The integral $\int_0^4 f(x) dx$ is estimated by the midpoint and trapezoid rules using ten subintervals. Which is the larger estimate, and are these estimates greater than or less than the actual integral? State your reasoning.