1. (8 points each) Compute the derivatives:
   
   (a) $f(x) = e^{7 \ln(x)} \ln(5e^x)$

   (b) $g(x) = \sin(2 \sin^{-1}(x))$

2. (10 points) Suppose $h(x) = x^5 + x + 2$.
   
   (a) Show that $h$ is one-to-one (so that it has an inverse function, $k$).

   (b) Compute $k'(4)$. 

Your name:
3. (8 points each) Compute the following:

(a) \( \lim_{x \to 0} \frac{\sin(x) - x}{x^3} \)

(b) \( \int \frac{2x}{x^2 + 5x + 4} \, dx \)

(c) \( \int_0^{\pi/2} (\sin(x))^2 (\cos(x))^3 \, dx \)

(d) \( \int x(\sec(x))^2 \, dx \)

(e) \( \int_1^2 \frac{dx}{x \ln(x)} \)
5. (16 points) A newly-discovered substance called Electionium has recently been dis- covered; a sample of 1758 g was found on November 8. As of November 30, this sample had decayed to 537 g. (a) Give an expression equal to the half-life $L$ of this substance. (b) When will this sample decay to less than 1 g?

It is acceptable to give either a numerical or a symbolic answer for each question. For example, answers such as \( L = \sin(3)/\ln(1) \) and \( 3L + 2 \text{ days after November 8} \) are in an acceptable form (but incorrect!).

6. (12 points) Find the length of the curve $y = \ln(\cos(x))$ from $x = 0$ to $x = \pi/4$. 
7. (16 points) The portion of the curve $y = 4 \arctan(x)$ between the points $(x, y) = (0, 0)$ and $(x, y) = (1, \pi)$ is rotated around the $y$-axis. Set up, but DO NOT EVALUATE, integrals which express the following:

(a) the surface area of the resulting surface

(b) the volume of the region enclosed by this surface and the plane at the top of it.

8. (10 points) A particle is moved along the $x$-axis by a force that measures $10/(1 + x)^2$ pounds at a point $x$ feet from the origin. Find the work done (in units of foot-pounds) to move the particle from the origin to a distance of 9 feet.
9. (8 points) Determine whether the sequence \( \{a_n\} \) converges or diverges, where \( a_n = (\ln(3n + 1) - \ln(2n + 5)) \). If it converges, compute \( \lim_{n \to \infty} a_n \), if not explain why not.

10. (10 points) Evaluate the sum \( \sum_{n=1}^{\infty} \left( \frac{3}{2^n} + \frac{5}{4^n} \right) \).

11. (10 points) Are there sequences of positive numbers \( b_n \) for which both series \( \sum_{n=1}^{\infty} b_n \) and \( \sum_{n=1}^{\infty} (1/b_n) \) converge? Give an example or explain why none exists.
12. (8 points each) State whether each of the following converges absolutely, converges conditionally, or diverges; explain why.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n}} \]

(b) \[ \sum_{n=1}^{\infty} \frac{2^n \sin(n)}{n!} \]

(c) \[ \sum_{n=1}^{\infty} \frac{n}{e^{n/8}} \]
13. (12 points) For which values of $x$ does the series $\sum_{n=1}^{\infty} \frac{(n - 1)2^n}{n!} x^n$ converge?

14. (16 points) (a) Compute the zeroth, first, second, and third terms of the Taylor series of $s(x) = \sqrt{1 + x}$ at $x = 0$.

(b) Use your series to estimate $\sqrt{1.1}$; you need not estimate the error in your approximation.

Congratulations! You survived to the end of a difficult course!