

Math 230 Final Examination May 7, 2002

No books, notes, or graphing calculators are allowed on this test. Your instructor will provide you with scratch paper, if you need it. Be sure that all of your work is shown and that it is well organized and legible. Each question is worth 8 points.

1. Euler's constant e is

(a) the number whose natural logarithm takes on the value 1

(b) the limit $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

(c) the sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$

(d) answers (a), (b), and (c) are all correct.

(e) none of the above answers is correct.

2. Let R be the region below the curve $y = \ln(x^2 + 1)$ and above the x -axis, $0 \leq x \leq 1$. Set up, but **do not evaluate**, an integral to compute the volume obtained by rotating the region R about the y -axis.

3. Set up, but **do not evaluate**, an integral to compute the length of the curve $y = \ln(x^2 + 1)$, $0 \leq x \leq \sqrt{3}$.

4. If $y = x^x$, find $\frac{dy}{dx}$.

5. Find $\frac{d}{dx} \ln \frac{\sqrt[4]{x-1} \sqrt[3]{1-x}}{\sqrt{x^2+1}}$. [Hint: laws of logarithms.]

6. Currently there are 500 tribbles aboard the U.S.S. Enterprise. If the tribble population increases at a rate of 50% per day, how long will it take to reach 20,000 tribbles? (You may leave your answer in logarithms.)

7. If $f'(x) = \frac{1}{\sqrt{x^3 + 1}}$, find $(f^{-1})'(y)$ evaluated at the value of y corresponding to $x = 2$.

8. Evaluate the integral $\int_0^{\pi/2} \cos^5(x) dx$.

9. Find the derivative of $e^{\tan^{-1}(2x)}$ with respect to x .

10. Integrate $\int \frac{\tan^2(\ln x)}{x} dx$.

11. Integrate $\int \frac{e^x dx}{1 + e^{2x}}$.

12. Integrate $\int \frac{dx}{(4 - x^2)^{3/2}}$.

13. Integrate $\int \sin^{-1}(3x) dx$. [Hint: Integrate by parts.]

14. Find the limit $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2}$.

15. What is wrong with the following use of l'Hospital's Rule:

$$\lim_{x \rightarrow 1} \frac{x^3 + x - 2}{x^2 - 3x + 2} = \lim_{x \rightarrow 1} \frac{3x^2 + 1}{2x - 3} = \lim_{x \rightarrow 1} \frac{6x}{2} = 3.$$

16. To integrate $\frac{x^3}{(x+1)^2(x-1)^2}$, you should write the integrand as

(a) $\frac{Ax^3}{x+1} + \frac{Bx^3}{(x+1)^2} + \frac{Cx^3}{x-1} + \frac{Dx^3}{(x-1)^2}$

(b) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

(c) $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-1} + \frac{Ex+F}{(x-1)^2}$

(d) $\frac{A}{(x+1)^2} + \frac{B}{(x-1)^2}$

(e) None of the above answers is correct.

17. Evaluate the improper integral $\int_0^{\infty} \frac{e^x}{1+e^{2x}} dx$.

18. Arrange the following five numbers in order, from smallest to largest:

$$100! \quad 2^{100} \quad \ln 100 \quad 100^2 \quad 100^{100}$$

19. Find the limit of the sequence $\{a_n\}$ if it exists (if not, say why):

$$a_n = \begin{cases} 1 - \frac{1}{n} & \text{if } n \text{ is even} \\ -1 - \frac{1}{n} & \text{if } n \text{ is odd.} \end{cases}$$

20. Evaluate the exact sum of the series $\sum_{n=0}^{\infty} \frac{e^{2n}}{2^{3n}}$.

21. Determine whether the following sum converges conditionally, absolutely, or diverges:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+2)!}{n! 3^n}$$

22. Determine whether the following sum converges conditionally, absolutely, or diverges:

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$$

23. Determine the values of x for which $\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n}$ converges.

24. Use the Taylor series expansion for cosine:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$

and the trig identity $\cos^2 x = \frac{\cos(2x) + 1}{2}$ to write $f(x) = \cos^2(x)$ as a power series.

25. Write the first 3 terms of the Taylor series centered at 0 for the function $f(x) = \sec x$.