1. Euler’s constant $e$ is
   (a) the number whose natural logarithm takes on the value 1
   (b) the limit $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$
   (c) the sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$
   (d) answers (a), (b), and (c) are all correct.
   (e) none of the above answers is correct.

2. Let $R$ be the region below the curve $y = \ln(x^2 + 1)$ and above the $x$-axis, $0 \leq x \leq 1$.
   Set up, but do not evaluate, an integral to compute the volume obtained by rotating
   the region $R$ about the $y$-axis.

3. Set up, but do not evaluate, an integral to compute the length of the curve $y = \ln(x^2 + 1)$, $0 \leq x \leq \sqrt{3}$. 
4. If $y = x^x$, find $\frac{dy}{dx}$.

5. Find $\frac{d}{dx} \ln \frac{\sqrt{x-1} \sqrt{1-x}}{\sqrt{x^2} + 1}$. [Hint: laws of logarithms.]

6. Currently there are 500 tribbles aboard the U.S.S. Enterprise. If the tribble population increases at a rate of 50% per day, how long will it take to reach 20,000 tribbles? (You may leave your answer in logarithms.)
7. If \( f'(x) = \frac{1}{\sqrt{x^3 + 1}} \), find \((f^{-1})'(y)\) evaluated at the value of \( y \) corresponding to \( x = 2 \).

8. Evaluate the integral \( \int_0^{\pi/2} \cos^5(x) \, dx \).

9. Find the derivative of \( e^{\tan^{-1}(2x)} \) with respect to \( x \).
10. Integrate \( \int \frac{\tan^2(\ln x)}{x} \, dx \).

11. Integrate \( \int \frac{e^x \, dx}{1 + e^{2x}} \).

12. Integrate \( \int \frac{dx}{(4 - x^2)^{3/2}} \).
13. Integrate \( \int \sin^{-1}(3x) \, dx \). [Hint: Integrate by parts.]

14. Find the limit \( \lim_{x \to 0} \frac{\sqrt{1 + x} - 1 - x/2}{x^2} \).

15. What is wrong with the following use of l’Hospital’s Rule:

\[
\lim_{x \to 1} \frac{x^3 + x - 2}{x^2 - 3x + 2} = \lim_{x \to 1} \frac{3x^2 + 1}{2x - 3} = \lim_{x \to 1} \frac{6x}{2} = 3.
\]
16. To integrate \( \frac{x^3}{(x + 1)^2(x - 1)^2} \), you should write the integrand as

(a) \( \frac{Ax^3}{x + 1} + \frac{Bx^3}{(x + 1)^2} + \frac{Cx^3}{x - 1} + \frac{Dx^3}{(x - 1)^2} \)

(b) \( \frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2} \)

(c) \( \frac{A}{x + 1} + \frac{Bx + C}{(x + 1)^2} + \frac{D}{x - 1} + \frac{Ex + F}{(x - 1)^2} \)

(d) \( \frac{A}{(x + 1)^2} + \frac{B}{(x - 1)^2} \)

(e) None of the above answers is correct.

17. Evaluate the improper integral \( \int_0^\infty \frac{e^x}{1 + e^{2x}} \, dx \).

18. Arrange the following five numbers in order, from smallest to largest:

\[
100! \quad 2^{100} \quad \ln 100 \quad 100^2 \quad 100^{100}
\]
19. Find the limit of the sequence \( \{a_n\} \) if it exists (if not, say why):

\[
a_n = \begin{cases} 
1 - \frac{1}{n} & \text{if } n \text{ is even} \\
-1 - \frac{1}{n} & \text{if } n \text{ is odd.}
\end{cases}
\]

20. Evaluate the exact sum of the series \( \sum_{n=0}^{\infty} \frac{e^{2n}}{2^3n} \).

21. Determine whether the following sum converges conditionally, absolutely, or diverges:

\[
\sum_{n=0}^{\infty} (-1)^n \frac{(n + 2)!}{n! 3^n}
\]
22. Determine whether the following sum converges conditionally, absolutely, or diverges:

\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}} \]

23. Determine the values of \( x \) for which \( \sum_{n=1}^{\infty} \frac{(x - 2)^n}{2^n} \) converges.
24. Use the Taylor series expansion for cosine:

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots
\]

and the trig identity \(\cos^2 x = \frac{\cos(2x) + 1}{2}\) to write \(f(x) = \cos^2(x)\) as a power series.

25. Write the first 3 terms of the Taylor series centered at 0 for the function \(f(x) = \sec x\).