

SHOW ALL WORK ON THESE SHEETS FOR FULL CREDIT

Name and ID: _____

1. (8 points each) Compute the derivatives of:

(a) $f(x) = e^{5 \ln(x^2+1)}$

(b) $g(x) = \sin^{-1}(\sqrt{1-x^2})$ (also written $g(x) = \arcsin(\sqrt{1-x^2})$.)

2. (10 points) Let $h(x)$ be the function defined by $h(x) = e^{2x} + 3x$.

(a) Show that h is one-to-one (so that it has an inverse function, $k = h^{-1}$).

(b) Compute $k'(1)$.

3. (8 points each) Compute the following:

(a) $\lim_{x \rightarrow 0} \frac{\tan(x) - x}{x^3}$

(b) $\int_0^{\pi/2} (\sin(x))^2 (\cos(x))^3 dx$

(c) $\int \frac{2}{x^2 - 8x + 7} dx$

(d) $\int x (\ln(x)) dx$

(e) $\int_{-\infty}^2 \frac{dx}{x^2 + 4}$

5. (16 points) Sketch the region of the plane bounded by the curve $y = 1 - x^3$, the vertical axis, and the line $y = -7$. Then, compute the volume of the solid formed by rotating the region around the y -axis.

6. (12 points) Find the length of the curve $y = \frac{2}{3}(1 + x^2)^{3/2}$ from $x = 0$ to $x = 3$.

7. (10 points) A particle is moved along the x -axis by a force that measures $5 + 4x$ pounds at a point x feet from the origin. Find the work done in moving the particle from the origin to a distance of 3 feet.

8. (16 points) The city of DeKalb had a population of 30,000 in 1980 and a population of 40,000 in 1990. Assuming the rate of population growth is proportional to the population at any given time, what is the expected population of DeKalb in 2010?

9. (8 points) Does the sequence $\left\{3 + \frac{\ln(n)}{n}\right\}$ ($n = 1, 2, 3, \dots$) converge? If so, to what? If not, why not?

10. (10 points) Evaluate the sum $\sum_{n=1}^{\infty} \left(\frac{3 + 2^n}{4^n}\right)$.

11. (10 points) Are there sequences of positive numbers b_n for which both series $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} (1/b_n)$ converge? Give an example or explain why none exists.

12. (8 points each) State whether each of the following converges absolutely, converges conditionally, or diverges; explain why.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(2n)}$$

$$(b) \sum_{n=1}^{\infty} \frac{3^n \cos(n)}{n!}$$

$$(c) \sum_{n=1}^{\infty} \frac{n}{e^{(\frac{n}{6})}}$$

13. (12 points) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(n-2)3^n}{n!} x^n$.

14. (16 points) (a) Compute the zero-th, first, second, and third terms of the Taylor series of $s(x) = \sqrt{1+x}$ at $x = 0$.

(b) Use your series to estimate $\sqrt{1.2}$. (You need not estimate the error in your approximation.)

That's it! Enjoy the summer; see you in Calculus III !