Math 230 Final Examination Name: ____________________________
May 9, 2006

No books, notes, or graphing calculators are allowed on this test. Your instructor will provide you with scratch paper, if you need it. Be sure that all of your work is shown and that it is well organized and legible. Each question is worth 10 points.

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1. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about the line $x = -2$.

2. Differentiate $y = \dfrac{e^{\sin x}}{x}$

3. Find $\dfrac{dy}{dx}$ if $y = x^{\tan^{-1} x}$.

4. Evaluate the limit $\lim_{x \to \infty} x^3 e^{-x^2}$

5. Bismuth–210 has a half–life of 5.0 days. A sample originally has a mass of 800 mg. Find a formula for the mass remaining after t days.

6. Integrate $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$

7. Integrate $\int_0^{1/2} \sin^{-1} x\,dx$

8. Integrate $\int \tan^3 x \sec x \, dx$

9. Integrate $\int \frac{x+5}{x^2-1} \, dx$

10. Evaluate the improper integral (if it converges): $\displaystyle\int_0^{\infty} \frac{1}{9+x^2}\,dx$

11. Assume f is a one–to–one differentiable function such that $f(2) = 9$ and $f'(2) = 4$. If $g = f^{-1}$, find (a) $g(9)$ and (b) $g'(9)$.

12. Set up, but do not evaluate, an integral for the length of the curve $y = \cos x$, $0 \le x \le 2\pi$.

13. Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{(\sqrt{2})^n + (\sqrt{3})^n}{(\sqrt{6})^n}$

14. Determine whether the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit: $a_n = \cos\left(\frac{2}{n}\right) + \frac{\sqrt{n}}{2+\sqrt{n}}$

15. Determine whether the following series converges absolutely, conditionally, or diverges: $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n\sqrt{\ln n}}$

16. Test for convergence: $\sum_{n=1}^{\infty} \frac{n!}{e^{(n^2)}}$

17. Test for convergence: $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

18. True or False?

(a) T F If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(b) T F If $0 < a_n \le b_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

(c) T F If $\displaystyle\sum_{n=1}^{\infty} |a_n|$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

(d) T F If $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = .95$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely.

(e) T F $\displaystyle\sum_{n=1}^{\infty} e^{1/n}$ diverges because the nth term does not go to 0.

19. Find the radius of convergence for $\sum_{n=1}^{\infty} \frac{x^n}{5^n\, n^5}$

20. Use the Maclaurin expansion for $\sin x$:

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

to evaluate the indefinite integral $\int \frac{\sin(x^2)}{x}$ as a power series. (Just give the first 5 terms of the series.)