Series Worksheet Hints for Selected Problems

Do not read these until you have given each problem a fair try. These hints will get you started. For many of the series on the worksheet, there are several ways to approach them.

Part I.

(a) Multiply by $\dfrac{1/n^{1000}}{1/n^{1000}}$.

(b) Multiply by $\dfrac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$.

(c) $|\sin n| \le 1$,

(d) Convert to fractional exponents then multiply by $\dfrac{1/n^{1/2}}{1/n^{1/2}}$.

(e) $\dfrac{n}{2n+1} \to \dfrac{1}{2}$ as $n \to \infty$.

(f) $\tan^{-1} n \to \dfrac{\pi}{2}$ as $n \to \infty$.

(g) Take ln of $\left(1+\frac{3}{n}\right)^{4/n}$.

(h) Similar to (g).

(i) Multiply by $\dfrac{1/4^n}{1/4^n}$ and use the fact that $\lim\limits_{x\to\infty} \dfrac{x^a}{b^x} = 0$ for pos. constants a and b.

(j) Multiply by $\dfrac{1/n}{1/n}$.

(k) Compute $\lim\limits_{n\to\infty} \dfrac{\ln(n+1)-\ln n}{1/n}$ by l'Hospital's Rule.

(l) Take ln of $n^{-\frac{1}{n}}$.

(m) We know by various simple arguments that $\dfrac{n+1}{n} \to 1$ as $n \to \infty$. (One way is to multiply by $\dfrac{1/n}{1/n}$; another way is to use l'Hospital's Rule.) What does the factor $(-1)^n$ do to the sign?

(n) $\left|(\frac{-2}{\pi}\right| < 1.$

Part II.

(a) geometric series, with first term = 1, ratio = $-\frac{1}{3}$.

(b) geometric series, with first term = $\frac{1}{8}$, ratio = $\frac{5}{8}$.

(c) Break the sum into two geometric series.

(d) Use partial fraction decomposition to write $\dfrac{1}{4n^2-1} = \dfrac{A}{2n-1} + \dfrac{B}{2n+1}$.

(e) limit comparison with $b_n = \frac{1}{n}$.

(f) See hint for (d).

Part III.

(a) Compare with the "p-series" $\sum \frac{1}{n^{2p-1}}$.

(b) Use Alternating Series Estimation Theorem on page 763.

Part IV.

(a) p-series with $p = \frac{1}{7}3$

(b) integral test

(c) Write $\ln\left(\frac{n+1}{n}\right) = \ln(n+1) - \ln(n)$ and examine the nth partial sum. (All but two terms will cancel,)

(d) split into two series $\sum_{n=1}^{\infty} \frac{2^n}{4^n+1} - \sum_{n=1}^{\infty} \frac{3^n}{4^n+1}$.

(e) limit comparison with $b_n = \frac{1}{n^{3/2}}$

(f) limit comparison with $b_n = \frac{1}{n}$

(g) compare with $b_n = \frac{1}{n^2}$

(h) compare with $b_n = \frac{1}{n}$

(j) nth term test

(k) alternating series test

(l) ratio test

(m) ratio test

(n) root test

(o) root test

(p) ratio test

(q) ratio test