MATH 232 FINAL EXAM Name (print) _____

Fall 2000 SSN _____

Show all your work.
No calculators are allowed.

1. **(12 points)** Consider the curve defined by the parametric equations $x = 1 - \cos t$, $y = t - \sin t$, $0 \leq t \leq 2\pi$.

   (a) Find an equation of the tangent line to the curve at $t = \dfrac{\pi}{2}$.

   (b) Set up (do not evaluate) an integral to find the length of the curve.

2. **(12 points)** Consider the curve defined by the polar equation $r = 1 + \cos\theta$.

   (a) Sketch the curve.

   (b) Set up (do not evaluate) an integral to find the area of the region enclosed by the curve $r = 1 + \cos\theta$.

3. **(10 points)** (*Modified from original exam.*) Let $f(x, y) = \dfrac{-xy + y}{(x-1)^2 + y^2}$. Find $\lim\limits_{(x,y)\to(1,0)} f(x, y)$ or show that the limit does not exist.

4. **(21 points)** Let $P(3,4,1), Q(1,7,1)$ and $R(-1,-2,3)$ be points in $\mathbb{R}^3$.

   (a) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$

   (b) Find an equation of the plane passing through the points P, Q, and R.

   (c) Find parametric equations for the line passing through the point $P(3,4,1)$ and perpendicular to the plane in (b).

5. **(15 points)** Let $f(x, y) = \ln(x - 2y)$.

   (a) Find $f_x(x, y)$ and $f_y(x, y)$.

   (b) Find the linear approximation $L(x, y)$ of f at $(5, 2)$.

   (c) Use $L(x, y)$ in (b) to approximate $f(5.2, 1.9)$.

6. **(12 points)** Let $f(x, y, z) = x^2 - 2y^2 + 3^2 + xyz$. Find the maximum rate of change of f at $(2, 1, 3)$, and the direction in which it occurs.

7. **(15 points)** Let $f_x(x, y) = y^2 + 4x$, and $f_y(x, y) = y(2x + 2)$.

(a) Find all critical points of f.

(b) Find the local maxima, local minima, and saddle points of f if there are any.

8. **(12 points)** Use the method of Lagrange multipliers to find the minimum value of $f(x, y, z) = 2x^2 + y^2 + 2z^2$ subject to the constraint $x - 3y - 2z = 23$.

9. **(16 points)**

(a) Evaluate

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\left(\sqrt{x^2+y^2}+1\right)dxdy.$$

(b) Given the integral $\int_0^2\int_1^{e^x} f(x,y)dydx$, sketch the region of integration and write an equivalent integral with the order of integration reversed. Do not evaluate the integral.

10. **(16 points)**

(a) Convert the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{z}{\sqrt{x^2+y^2+z^2+1}} dzdydx$ into an integral in spherical coordinates. Do not evaluate the integral.

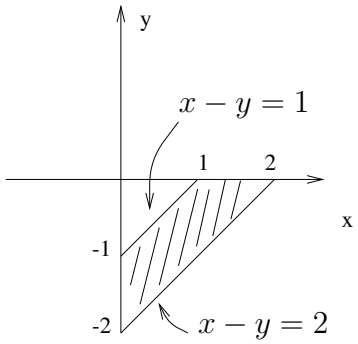
(b) Convert the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{z}{\sqrt{x^2+y^2+z^2+1}} dzdydx$ into an integral in cylindrical coordinates. Do not evaluate the integral.

11. **(10 points)** Find the mass of a thin wire in the shape of the circular helix $\vec{\gamma}\,(t) = 2\cos t\ \vec{i}\ + 2\sin t\ \vec{j} \rightarrow + t\ \vec{k}, 0 \leq t \leq \pi$, if the density function is $\rho(x, y, z) = y + z$.

12. **(14 points)**

(a) Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ of the transformation $x = \dfrac{1}{2}(u+v)$, $y = \dfrac{1}{2}(u-v)$.

(b) Use your answer to (a) to evaluate the integral which is the surface area of the part of the plane $z = 2x + 3y + 10$ that lies above the trapezoidal region R with vertices $(1,0), (2,0), (0,-2)$, and $(0,-1)$.



13. **(20 points)** Consider the vector field $\vec{F}(x,y) = (1+ye^{xy})\ \vec{i} + (2y+xe^{xy})\ \vec{j}$ in $\mathbb{R}^2$.

(a) Show that $\vec{F}$ is conservative.

(b) Find a function f such that $\bigtriangledown f = \vec{F}$.

(c) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is a smooth curve given by $\vec{r}(t) = te^t\ \vec{i} + (1+t)\ \vec{j}, 0 \leq t \leq 1$.

14. **(15 points)**

(a) State Green's Theorem.

(b) Use Green's Theorem to evaluate the line integral $\oint_C y^2 dx + 3xy dy$, where C consists of the curve $y = x^2$, $0 \leq x \leq 2$, and the line segment from $(2, 4)$ to the origin $(0, 0)$.