1. **(8 pts.)** Set up, but do not evaluate an integral that represents the length of the curve $C$ given parametrically by $x(t) = t + \cos t$, $y(t) = t - \sin t$ for $0 \leq t \leq 2\pi$.

2. **(10 pts.)** Use the cross product of two vectors to determine the area of a parallelogram for which three vertices are $P(1, 4, 6)$, $Q(-2, 5, -1)$ and $R(1, -1, 1)$.

3. **(8 pts.)**
   
   (a) Describe geometrically the trace of the quadric surface $16x^2 = y^2 + 4z^2$ in the plane $z = 1$.

   (b) Describe geometrically the trace of the quadric surface $x^2 - y^2 + z^2 = 1$ in the plane $y = 1/3$. 

There are 200 total points on this examination.

**SHOW YOUR WORK!**
4. (10 pts.) Find the equations of the tangent plane and the normal line at the point $(-2, 1, -3)$ to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

5. (14 pts.) For each of the following, either show that the limit exists and find the limit, or show that the limit does not exist.

   (a) \[ \lim_{(x,y) \to (0,0)} \frac{4x^2y}{x^2 + y^2} \]

   (b) \[ \lim_{(x,y) \to (0,0)} \frac{(xy)^2}{x^4 + y^4} \]

6. (12 pts.)

   (a) Let $f(x, y) = \ln(x + y^2) + \sin x$. Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
(b) Let $f$ be as in (a). Suppose that $x = se^t$ and $y = 1 + se^{-t}$. Find $\frac{\partial f}{\partial t}$.

7. (14 pts.) Let $f(x, y) = 2x^2 - 4xy + y^4 + 2$.

(a) Find all critical pts. of $f$.

(b) Find the local maxima, local minima, and saddle points for $f$ if there are any.
8. (12 pts.) Use the method of Lagrange Multipliers to find the maximum value of $f(x, y) = 4x^3 + y^3$ subject to $2x^2 + y^2 = 1$

9. (a) (10 pts.) Set up a double integral to find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(b) (12 pts.) Sketch the region of integration for the integral $\int_0^2 \int_{y/2}^1 e^{x^2} \, dxdy$ and evaluate the integral by reversing the order of integration.
(c) **(8 pts.)** Express the integral \( \int_{1/\sqrt{2}}^{1} \int_{\sqrt{1-x^2}}^{x} \frac{1}{x^2 + y^2} \, dy \, dx \) as a double integral in polar coordinates.

10. **(10 pts.)** Let \( G \) be the wedge in the first octant that is cut from the cylindrical solid \( y^2 + z^2 \leq 1 \) by the planes \( y = x \) and \( x = 0 \). Evaluate

\[
\iiint_{G} z \, dV.
\]

11. **(14 pts.)**

(a) Find the Jacobian \( \frac{\partial(x, y)}{\partial(u, v)} \) of the transformation \( x = \frac{1}{2}(u + v), \quad y = \frac{1}{2}(u - v) \).

(b) Use your answer in (a) to re-express the integral \( \iint_{R} \frac{x - y}{x + y} \, dA \) as an iterated integral in terms of the variables \( u \) and \( v \) where \( R \) is the region enclosed by \( x - y = 0, \ x - y = 1, \ x + y = 1 \) and \( x + y = 3 \).
12. **(a) (8 pts.)** Set up, but do not evaluate, an integral that gives \( \int_C 2z \, ds \) where \( C \) is the curve defined by \( \vec{r}(t) = t^2 \hat{i} + t^4 \hat{j} + t^6 \hat{k} \) when \( 0 \leq t \leq 3 \).

**(b) (20 pts.)** Consider the vector field \( \vec{F}(x, y) = 2xy^3 \hat{i} + (1 + 3x^2y^2) \hat{j} \) in \( \mathbb{R}^2 \).

i. Prove that \( \vec{F} \) is conservative.

ii. Find a function \( f \) such that \( \nabla f = \vec{F} \).

iii. Evaluate the line integral \( \int_C \vec{F} \cdot d\vec{r} \) where \( C \) is given by \( \vec{r}(t) = (t + 1) \hat{i} + (2 - t^2) \hat{j} \) when \( 0 \leq t \leq 2 \).
13. **(14 pts.)** Suppose you are climbing a hill whose shape is given by the equation
\[ z = 1000 - 0.01x^2 - 0.02y^2 \]
where \(x, y,\) and \(z\) are measured in meters, and you are standing at a point with coordinates \((50, 80, 847)\). The positive \(x\) axis points east and the positive \(y\) axis points north.

(a) If you walk northwest, will you start to ascend or descend? At what rate?

(b) In which direction is the slope largest? Why?

14. **(16 pts.)**

(a) Determine the sets of points on the surface \(z = f(x, y) = 12 - |x - 5|\) where the function \(f\) is NOT differentiable. Explain.
(b) A table of values is given for a function $f(x, y)$ defined on $R = [1, 3] \times [0, 4]$. Estimate $\iint_R f(x, y) \, dA$ using the Midpoint Rule with $m = n = 2$. 

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