FINAL EXAM Name (print) ______________

MATH 232 ZID ______________

Spring 2006 Signature ______________

Show your work !

1. (7 points) Consider the curve C defined by $\vec{r}\ (t) = 2t\ \vec{i}\ + ln\ t\ \vec{j}, 1 \leq t \leq e$. Set up, but do not evaluate, an integral to compute the length of C.

2. (7 points) Use the cross product to find the area of the triangle in 3-dimensional space with vertices $(0, 0, 0), (2, 3, -1)$ and $(3, -1, 4)$.

3. (8 points) Find the position vector for an object at any time t which has acceleration $\vec{a}\ (t) = < 6t, 12t + 2, e^t >$, initial velocity $\vec{v}\ (0) = < 2, 0, 1 >$, and initial position $\vec{r}\ (0) = < 0, 3, 5 >$.

4. (8 points) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be nonzero vectors in these dimensional space. For each statement below either provide a specific counterexample, or explain why the statement is always true.

(a) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then $\vec{b} = \vec{c}$.

(b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

5. (16 points) (a) Find the equation of the plane containing the point $(2, 1, -1)$ and parallel to the plane $3x - y + 2z = 1$.

(b) Determine the tangent plane and normal line to the surface $z = x^3 + y^3 + x^2/y$ at $(2, 1, 13)$.

6. (12 points) (a) Compute $\dfrac{\partial^2 f}{\partial x^2}$ for $f(x,y) = e^{xy} + x\sin y$

(b) Use the chain rule to compute $\dfrac{\partial f}{\partial u}$ when $f(x,y) = 4x^2y^3, x = u^3 - v\sin u$ and $y = 4u^2$.

7. (16 points) (a) Evaluate the following limit or show it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2xy^3}{x^2+8y^6}.$$

(b) Sketch in the plane the domain of the function $f(x,y) = ln(3 - x^2 + y)$, and determine all points at which f is continuous.

8. (14 points) Let $f(x, y) = 3x - x^3 - 3xy^2$.

   (a) Explain why $(1, 0)$ is a critical point for f.

   (b) Determine if f has a local maximum, local minimum, or saddle point at $(1, 0)$.

9. (8 points) Suppose the elevation on a hill is given by $f(x, y) = 200 - y^2 - 4x^2$. From the site at $(1, 2)$, in which direction will the rain run off?

10. (a) (12 points) Set up a polar double integral to find the area of one petal of the rose described by the equation $r = \sin 5\theta$.

(b) Express the integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} x^2(x^2+y^2)^2 dy\ dx$ in polar coordinates. Do not evaluate.

11. (12 points) Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = 4x^2y$ subject to the constraint $x^2 + y^2 = 3$.

12. (24 points) (a) Evaluate $\int_1^2 \int_0^{2/x} e^{xy} dy dx$

(b) Express $\int_1^2 \int_0^{ln\ y} f(x,y) dx\ dy$ as an equivalent integral with order of integration reversed.

(c) Set up an integral to find the volume of the solid bounded by $z = 6 - x - y, z = 0, x = 4 - y^2,$ and $x = 0$.

13. (14 points) (a) Determine the limits (but not the value of the integral) for the triple integral $\int\int_Q\int (6xy) dz\ dy\ dx$ where Q is the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $2x + y + z = 4$.

(b) Set up, but do not evaluate, a triple integral in spherical coordinates to find the volume of a solid in the first octant that is bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$.

14. (12 points) (a) Calculate the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$ for the transformation $x = \dfrac{1}{5}(u - v), y = \dfrac{1}{5}(2u + 3v)$.

(b) Use part a) to re-express $\displaystyle\int_R\int \frac{e^{y+3x}}{y - 2x} dA$ as an iterated integral in terms of the variables u and v, where R is the region bounded by $y = 2x - 1, y = 2x + 5, y = 1 - 3x$, and $y = -1 - 3x$.

15. (16 points) (a) Compute the work done by the force field $\vec{F}(x,y) =< y, -x >$ acting on an object as it moves along the parabola $y = x^2 - 1$ from $(1,0)$ to $(-2,3)$.

(b) Evaluate $\int_C (3x + y)ds$ where C is the line segment from $(5,2)$ to $(1,1)$.

16. (14 points) Choose one of the following:

(a) Prove the vector field $F(x,y) =< 3x^2y^2, 2x^3y - y >$ in R^2 is conservative, and find f so $\nabla f = F$.

or

(b) Use Green's Theorem to evaluate $\int_C (e^x + y^2)dx + (e^y + x^2)dy$ where C is the positively oriented boundary of the region in the first quadrant bounded by $y = x^2$ and $y = 4$.