1. Find the general solution of the system of equations

\begin{align*}
2x - 4y + z + 6w &= 0 \\
3x - 6y + 6z + 15w &= -3 \\
x - 2y - z + w &= 1
\end{align*}

2. Find the determinant

\[
\begin{vmatrix}
0 & -3 & 4 & -1 \\
0 & 0 & 1 & -2 \\
2 & 5 & -3 & 1 \\
0 & 0 & 0 & 4
\end{vmatrix}
\]
3. Suppose $L : \mathbb{R}^3 \to \mathbb{R}^3$ is such that

$$L \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + 2y - z \\ 2x + y + 4z \\ 3x - y + 3z \end{bmatrix}$$

(a) Find a basis for ker $L$.

(b) Find a basis for range $L$. 

4. (a) Are the vectors \[
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix},
\begin{bmatrix}
2 \\
3 \\
0
\end{bmatrix}, \text{ and } 
\begin{bmatrix}
1 \\
2 \\
-1
\end{bmatrix}
\] independent?

(b) Is the matrix \[
\begin{bmatrix}
1 & 2 & 1 \\
1 & 3 & 2 \\
1 & 0 & -1
\end{bmatrix}
\] invertible? Explain.

5. Let \( P_2 \) denote the collection polynomials of degree \( \leq 2 \) and the 0 polynomial. Define \( L : P_2 \to \mathbb{R}^2 \) by
\[
L(p) = \begin{bmatrix}
p(1) \\
p(2)
\end{bmatrix}.
\]

Let \( S = \{1, 2 + t, 1 + t + t^2\} \), a basis of \( P_2 \) and \( T = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \), a basis for \( \mathbb{R}^2 \).

Find the matrix of \( L \) for the bases \( S \) and \( T \).
6. Let \( \mathbb{R}^4 \) have the usual (dot) inner product. Find an orthonormal basis for the subspace spanned by \( \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\} \).

7. Let \( A = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 4 \\ 0 & 4 & 0 \end{bmatrix} \).

(a) Find the eigenvalues of \( A \).

(b) Find a matrix \( P \) so that \( P^{-1}AP \) is diagonal.
8. (a) Let $A$ and $B$ be $n \times n$ matrices. Define “$A$ is similar to $B$”.

(b) Prove that if $A$ is similar to $B$ and $B$ is similar to $C$, then $A$ is similar to $C$.

(c) Suppose that $A$ is similar to $B$ and $\lambda$ is a scalar. Show that $A - \lambda I$ is similar to $B - \lambda I$.

9. Suppose that $v_1$ and $v_2$ are linearly independent eigenvectors of a matrix $A$ with corresponding eigenvalues $\lambda_1$ and $\lambda_2$. If $\lambda_1 \neq \lambda_2$, show $v_1 + v_2$ is not an eigenvector of $A$. 

10. Show that if an $n \times n$ matrix $A$ satisfies $A^T A = I$, then $\det(A) = \pm 1$.

11. Let $V$ and $W$ be vector spaces and $L : V \to W$ a linear map. Assume $S = \{v_1, ..., v_n\}$ is a basis of $V$ and set $T = \{L(v_1), ..., L(v_n)\}$.

(a) Show that if $T$ spans $W$, then $L$ is onto.

(b) Show that if $L$ is 1–1, then $T$ is independent.
12. Let $W$ be a subspace of the inner product space $V$. Define

$$W^\perp = \{ v \in V \mid (v, w) = 0 \text{ for all } w \text{ in } W \}.$$  

(a) Show that $W^\perp$ is a subspace of $V$.

(b) Show that the only vector in both $W$ and $W^\perp$ is 0.

13. Let $V$ be an inner product space. Suppose that $\{v_1, v_2\}$ is an orthonormal set. Show

$$||c_1v_1 + c_2v_2||^2 = c_1^2 + c_2^2$$