Math 240

Final Exam

Fall 2008

1. (15) Find the general solution of the system of equations

$$\begin{array}{rrrrcr} x & -3y & & +2w & = & 4 \\ -2x & +6y & -3z & +w & = & 1 \\ x & -3y & +2z & & = & -2 \\ 2x & -6y & +z & +3w & = & 5 \end{array}$$

2. (15) The inverse of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ is $\begin{bmatrix} 0 & -1 & 3 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$. Use this information to solve the system

$$\begin{array}{rrrcr} x & -2y & +3z & = & 4 \\ 2x & -3y & +3z & = & -2 \\ x & -y & +z & = & 5 \end{array}.$$

3. (15) Suppose that A and B are $n \times n$ matrices with $A^{-1} = A^T$ and $B^{-1} = B^T$. Show that $(AB)^{-1} = (AB)^T$.

4. (15) Suppose that $L : V \to W$ is an onto linear map and that $\{v_1, \cdots, v_n\}$ spans V. Show that $\{L(v_1), \cdots, L(v_n)\}$ spans W.

5. (15) In P_1 with the inner product $(p, q) = \int_0^1 p(t)q(t)\ dt$, find $\|p\|$, $\|q\|$, and $\cos\theta$ where θ is the angle between $p(t) = 2t + 3$ and $q(t) = t - 1$.

6. (20) The reduced row echelon form for the matrix $A = \begin{bmatrix} 1 & -2 & 1 & 7 \\ -1 & 2 & 0 & -3 \\ 2 & -4 & -3 & -6 \\ 1 & -2 & 1 & 7 \end{bmatrix}$ is

$$\begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) What is a basis for the null space of A?

(b) What is a basis for the column space of A?

(c)

7. (20) Define $L : P_2 \to \mathbb{R}^2$ by $L(p) = \begin{bmatrix} p(1) \\ p(0) \end{bmatrix}$. Let $S = \{t^2 + 2t - 1,\ t + 3,\ 1\}$ and $T = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

(a) Suppose the coordinate vector $[p]_S = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$. Find p.

(b) Find $A = [L]_{T \leftarrow S}$, the matrix representation for L with respect to the ordered bases S and T.

8. Show that $\|v+w\|^2 + \|v-w\|^2 = 2\|v\|^2 + 2\|w\|^2$.

9. (20) Let $A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$. Find a matrix P so that $P^{-1}AP$ is diagonal.

10. (20) The characteristic polynomial of the matrix $A = \begin{bmatrix} 2 & -2 & 1 \\ -2 & 6 & -3 \\ -4 & 8 & -4 \end{bmatrix}$ is $\lambda(\lambda - 2)^2$.

(a) Find the eigenvalues of A.

(b) For each eigenvalue, find corrsponding eigenvectors of A.

(c) Is A diagonalizable? Justify your answer.

11. (15) Find the characteristic polynomial and the eigenvalues of the matrix $\begin{bmatrix} 3 & 2 & -1 \\ -6 & -1 & 3 \\ 2 & 4 & 0 \end{bmatrix}$.

12. (15) Let W be the span of the vectors $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$.

(a) Find an orthonormal basis for W.

(b) Find an orthonormal basis for $W^{\perp}$.