Pythagorean Theorem – Euclid’s Proof

Given a right triangle \( \triangle ABC \). Construct a point \( D \) on the line \( \overrightarrow{BC} \) such that \( m(\angle CAD) = m(\angle B) \), as shown in the following diagram.

The following three right triangles are similar:
\[ \triangle ABC \sim \triangle DAC \sim \triangle DBA. \]

It follows that
\[ \frac{a}{c} = \frac{c}{a + d} \]

So
\[ a^2 + ad = c^2 \]

Also
\[ \frac{a}{b} = \frac{d}{b} \]

So
\[ ad = b^2 \]

Thus
\[ a^2 + b^2 = c^2. \]