

Name _____

Math 360
Final Exam
Spring 2011

1. A railroad system like the one described in class has stations at $x_1 = 0, x_2 = 4, x_3 = 6, x_4 = 9, x_5 = 14, x_6 = 20$. The number of resupply trips per year they require are $N_1 = 6, N_2 = 2, N_3 = 7, N_4 = 3, N_5 = 6, N_6 = 7$, respectively.

(a) What is the optimal location of the parts distribution center?

- (b) Suppose the station located at $x_6 = 20$ reduces the number of resupply trips needed. At what value of N_6 should the parts distribution center be moved to a new location?

2. A rat is twice as long as a mouse. It has been determined that the rate of heat loss of an animal is proportional to its surface area. The rate of heat generation is proportional to the product of its volume and its metabolism rate, R . If heat generation and heat loss are to be balanced, how should the metabolism rate of a rat compare to that of a mouse? Give a value of the ratio R_{rat}/R_{mouse} .

3. It is desired to fit the data $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ to a curve of the form $y = Ax^2 + B$. Use the least squares method to find the optimal values of A and B .

4. The force F of a spring extended by a distance x is given by $F = -kx$. Given that $F = ma$ where m is a mass and a is an acceleration,

(a) Find the dimensions of the constant k

(b) If t is the time it takes for one oscillation of the spring, use dimensional analysis to find an expression for t in terms of m, k, ℓ , the initial length of the spring, and g , the acceleration due to gravity $[g] = LT^{-2}$.

5. A system is described by the differential equation $\frac{dy}{dx} = -y^2 + 7y - 10$, $y(0) = y_0$.

(a) What are the equilibrium values for y ? Classify them as either stable or unstable equilibria.

(b) Determine the values of y giving inflection points and determine the concavity of the solutions.

(c) Use the information above to sketch several solutions of this initial value problem.

(d) Suppose we have the initial condition $y(0) = 3$. What is the limiting value of y as $x \rightarrow \infty$?

6. The population of hawks, x and of eagles, y satisfies the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= .0004x(200 - y) \\ \frac{dy}{dt} &= .0002y(150 - x)\end{aligned}$$

Determine the equilibrium values of x and y and draw a phase diagram in the $x - y$ plane for this system. What do you predict will happen if the initial population of hawks is 170 and the initial population of eagles is 150?

7. Find the solution of $\frac{dy}{dx} = (1+x)(1+y^2)$ such that $y(0) = 3$.

8. Find the maximum value of $f(x, y) = 2x + y$ subject to the conditions

$$\begin{aligned}x, y &\geq 0 \\x + y &\geq 8 \\x &\leq 10 \\y &\leq 12 \\2x + 3y &\leq 36\end{aligned}$$