Adjusting the Predator-Prey model

In this project we look at some variations to the discrete Predator-Prey model discussed in class. The basic model we start from will have the population of mice, x_n, follow the relation $\Delta x_n = r_1 x_n(1 - \frac{y_n}{M_1})$ and the population of hawks, y_n will satisfy $\Delta y_n = r_2 y_n(1 - \frac{x_n}{N_1})$

1. If the population of mice grows at a 5% rate per generation in the absence of hawks, and the population of hawks falls by 6% per generation in the absence of mice, what can you say about the parameters in this model?

2. Suppose that there is an equilibrium when the population of mice is $200,000$ and that of hawks is 1500. What can you say about the parameters of this model?

   Now assume that the conditions in the last two problems hold. This should determine all of the parameters.

3. If the initial population of mice is $300,000$ and the initial population of hawks is 1200, how do the two populations change over time? Consider this over the course of at least 500 generations. How long does it take for the scenario to 'cycle'? What happens to the populations from one cycle to the next? Do they change by a constant ratio? What do you think the very long term behavior of this model will be?

4. Continuing from the previous problem, what is the population of hawks when the population of mice is a local maximum? A minimum? How about the population of mice when the hawks are at a maximum or minumum? Can you explain why this happens?

5. If, instead, the initial population of mice was $150,000$ and that of hawks was 1600, how do your answers differ from the previous case?

   We now modify our model to take into account the fact that the land can't accomodate an unlimited number of either species. To do this, we include an additional factor limiting the growth of each population as follows. We let $\Delta x_n = r_1 x_n(1 - \frac{y_n}{M_1})(1 - \frac{x_n}{N_2})$ and similarly, $\Delta y_n = r_2 y_n(1 - \frac{x_n}{N_1})(1 - \frac{y_n}{M_2})$

6. Explain the significance of N_2 and M_2. Why would we expect to have $N_1 < N_2$ and $M_1 < M_2$?

7. Take $N_1 = 400000$ and $M_2 = 3500$. Replay the scenarios above from questions $3, 4$ and 5. What do you notice? How could this be useful?

8. Explore what happens if the initial populations are above N_2 or M_2. How many cases does this give? You might wish to limit the number of generations in these cases. Can you explain this behavior from our model? Is this realistic?