In this project we model the speed of a competitive swimmer when she is engaging in a long lap drill doing the front crawl, e.g. a 4000 meter drill. Since it is a long-distance swim and does not involve any spinning motion, the swimmer would just proceed smoothly in water with a uniform speed.

In this part of the project, we divide the model into two submodels, the submodel for the propulsion force $F$ as generated by the strokes and the kicking of the swimmer, and the submodel for the water resistance force $R$. When the swimmer moves at a constant speed, there is no acceleration; by one of Newton’s laws of motion, that means there is no net force acting on the swimmer’s body, i.e. $F$ and $R$ are equal.

The submodel for $F$ uses several variables:

i. $P$, the power of the swimmer, i.e. the amount of mechanical work that he can perform per unit time.

ii. $n$, the number of strokes he can complete per unit time (sometimes called his stroke rate).

iii. $g$, the gravitational acceleration on earth. (This will be relevant because gravity affects the surface waves generated by the swimmer’s kicks, and it is harder to generate forward force $F$ when swimming in turbulent water.)

iv. $\mu$, the viscosity of the water. (Viscosity is essentially a measurement of the cohesiveness of the water molecules — how tightly they are attached to each other — and thus the higher the viscosity, the more the water will resist the swimmer’s kicks, making the kicks more effective.)

v. $\ell$, the length of the swimmer. (A professional swimmer will always finish his underwater strokes all the way past his hip, trying to extend his arms as far back as possible. As a result, the longer the body length of the swimmer, the further backward he would be able to push with each stroke, thus increasing propulsive force $F$.)

Step 1 Catalogue the dimensions of all the variables in the variable set and tabulate the dimensions in a table.
Step 2 Using dimensional analysis, show that the propulsion $F$ will be given by a formula of the form

$$F = \ell^{3/2} g^{1/2} \mu \phi \left( \frac{P}{g \mu \ell^2}, n, \frac{\ell}{g} \right)$$

(1)

for some function $\phi$ of three variables.

Step 3 Now we turn to the submodel for $R$, the force of water resistance on the swimmer. In this case $R$ depends on

i) $A$, the cross-sectional area of the swimmer, since the clearly a smaller cross-section offers less resistance to movement through water

ii) $v$, the velocity of the swimmer, since there is more resistance when traveling faster through a fluid

iii) $\rho_w$, the density of the water

iv) $\mu$, the viscosity of water. In this case, unlike the first submodel, $\mu$ plays a suppressive role on the motion of the swimmer: the more cohesive the water particles are, the stronger the resistance $R$ would be when the swimmer tries to break through by streaming forward.

As in the first submodel, using dimensional analysis show that $R$ will be given by a formula of the form

$$R = A v^2 \rho_w f \left( \frac{\mu}{\rho_w v \sqrt{A}} \right)$$

(2)

for some function $f$ of one variable.

Step 4 Suppose it has been shown experimentally that $R$ is proportional to $\mu^{2/3}$ (when all other variables are held constant). Show that in this case we have

$$R = k A^{2/3} \rho_w^{1/3} v^{4/3} \mu^{2/3}$$

(3)

for some (dimensionless) constant $k$. Using this formula, would an increase in $A$ increase or decrease the force of water resistance? Does this seem physically reasonable? Answer likewise for $v$.

Step 5 Combining (1) and (3), derive a formula for $v$, the speed of the swimmer, assuming she is not accelerating (cf. the project introduction)

Step 6 Suppose that, through experimental data, the function $\phi$ above is found to be increasing in both its variables, that is, $\phi(\Pi_1, \Pi_2)$ increases when either of $\Pi_1$ or $\Pi_2$ is increased. Suppose further that it is determined that the function $\phi(\Pi_1, \Pi_2) = h (\Pi_1)^{\alpha} \Pi_2$ for some constant $h$ and $\alpha > 0$. Moreover suppose that longer swimmers have lower stroke rates, embodied by the relationship $n \propto \ell^{-1/2}$.

How would $v$ depend on each of $P, n, A, g, \mu$ and $\ell$?
Step 7 As we have already observed, a higher viscosity $\mu$ increases $F$ in the first submodel (which helps a swimmer go faster) but also increases $R$ in the second one (which works against the swimmer). The interesting thing would be to figure out the overall effect of $\mu$ on the motion of the swimmer. Using your results in Step 6, what would be your conjecture about the overall effect of $\mu$ on $v$? That is, does a higher viscosity help or hinder the swimmer?