MATH 402 Methods of Instruction in the Mathematics Curriculum for the Elementary School
Course Outline - Dr. Alan Zollman

Introduction

Do "Rolling Rectangle" Activity (Sample Problem-Solving Presentation)
What is the math topic?
Is this a good math lesson? Why?
What is the role active student involvement? -manipulatives? -prediction? -reflection? -connection?

Read Aloud <u>The Math Curse</u> by Scieszka & Smith

Teaching Mathematics

Story Problems done as a class

Describe Philosophy of Mathemtics Education Paper Draft of six paragraphs
Write answers to the six questions:

1. What is the purpose of schools/education?
2. What are the role and responsibility of the student?
3. What are the role and responsibility of the teacher?
4. What is the purpose of learning mathematics?
5. What are effective learning strategies?
6. What are effective teaching strategies?

Assignment: List the "primary" (D'Neilian) numerals from 1 to 21
Write the correct spelling for the numbers from 1 to 21
Find "Real-World" Numbers

Learning vs. Teaching

List 6 characteristics you would use to evaluate a good math lesson
Watch the Marshmallow Video of 2nd Grade Division
What are the positives, the negatives in the lesson?

What are the characteristics of a good math teacher?
What types are these characteristics?
How does ***"affect" of the teacher*** (e.g., caring, good listener) fit into teaching math?
How does ***knowledge of mathematics*** fit into teaching math?
How does ***knowledge of mathematics education*** fit into teaching math?

Lesson Planning:

- Mathematical Conceptual Conent Development
- Piagetian Order-of-Presentation Development
  (real world example to concrete/manipulative to pictorial to symbolic back to real world application)
- Lesson Plan Development
  (Objective, Motivation, Individual/Cooperative Group Work, Closure, Assessment)

Mathematical Problem Solving

NCTM Process Standards vs. the Content Standards

Do Knowing vs. Understanding Activity
What is the difference between Correct Understanding and Correct Answers?
What should we teach for in mathematics?
What is the difference between conceptual and procedural understandings?
What is the difference between learning math and teaching math?
In what order, according to educational research, do children ***learn*** math?
Describe <u>*Teaching Children Mathematics*</u> critique

Number Sense

Do "attribute pieces" activity
What attributes do students see in objects?
How do you define the color RED?
How do you define the number FOUR?
How is number like color, size, shape, texture, weight? How not?
What is a number?
What is a numeral?
What are the uses of numbers in the real world?
What does Cardinal, Ordinal, Nominal mean?

How does a student get meaning from counting?
Why are there different methods (e.g., "point counting") of counting?

When is 3 greater than 4?
When is 3 equivalent to 4?
When is 4 greater than 3?
Why should we say "greater than" as oppose to "more than" in finite quantities vs. mass?
Why should we say "fewer than" as oppose to "less than" in finite quantities vs. mass?
What really is meant by "greater than" and "fewer than?"
What is the purpose of number & name in numeration?

Place Value, Numeration

Do popcycle sticks and rubber bands activity
Ask students to count their sticks -- in ***base four***
Why are ***we*** doing base four instead of base ten?
How many numbers are in base four?
Why is there not "4" in base four?
After we get through "0". "1", "2", "3", then what's next?
Why do we ***group***? How do we group?
After we get "1" group, what comes next in our counting?
What does "11" really mean? Why do we write this as ***"1 group and 1 single"***?
How is"12" is different than "21"?
What happens if a student does not know why "12" is different than "21"?
What is the biggest two-digit number in base four?
What comes next after ***"33"***?

Join ***(add)*** your sticks to the set of sticks of your partner
How many do you have now?
Get, as partners,"13" sticks and add to it "12" sticks -- what do you have to do?
Why is this different than "12" plus "21"?
Why is this ***(grouping)*** a ***big deal conceptually*** for students?
Why is it important to have students precisely say ***number and group name***?
Why should we not be asking students to write the exercise down on paper now?
Why should we not be doing "work sheets" now?
Get, as partners,"102" sticks and take away ***(subtract)*** "21" sticks
Why is this different than "32" minus "21"?
What happens in "102" minus "23"?
Why is this ***(regrouping)*** a ***big deal conceptually*** for students?
Why is it important to have students precisely say ***number and group name***?
Why should we not be asking students to write the exercise down on paper now?
Why should we not be doing "work sheets" now?

In base ten, give ten "nicknames" (equivalent numerals) for 24 sticks
How does a place-value expander work?
What is meant by ***expanded notation***?
How is counting, adding, grouping, subtracting, and regrouping the sticks all connected conceptually?

Addition of Whole Numbers

When do we ***add*** in the real world? Give a situation for each of the two models.
Is addition always either "joining" of common elements of sets or "measurements?"

When is 3 plus 4 equivalent to 7? When is 3 plus 4 equivalent to 51?
In base ten, what would be equivalent to 3 tens plus 4 ones?
What do we have to know ***before*** we add the numbers?

Do "memory strategy" activity of listing 12 non related nouns with 60 seconds to remember them all
How many could you list? Which ones?
If people's "M-Space" is 5 to 7 items (± 2), how can people remember 10 digit phone numbers?
What conceptual strategies do we use instead of just drill and memorization?
Do another "memory strategy" activity of 12 nouns, but make a whole story to connect the words
How might we use this with elementary students?
Can we substitute a conceptual strategy for a procedural method?

Do students need to know their basic addition facts, e.g., 5 + 7?
How many basic addition facts are there?
Of the 100 addition facts, how many do students really need to know?
How does the commutative property for addition fit into this idea?
How does the associative property for addition fit into such problems as 5 + 8 (making tens)?
What about doubles, and near-doubles (5 + 6) strategies?

How will we add numbers greater than 9 + 9?
Do place value charts and popcycle sticks with rubber bands in base six.
Add 14 base six + 2 5 base six.
What concept is important? Why is ***grouping*** the "key?"
Why are 75% of students' add, subtract, multiply, and divide algorithms errors really place value errors?

Why do we transfer for manipulatives (popcycle sticks) to a numeral representation?
How do we transfer for manipulatives (popcycle sticks) to a numeral representation?
Show 14 base six + 2 5 base six in expanded notation symbolically,

e.g.,	*1 group*	*4 singles (base six)*
+	*2 groups*	*5 singles (base six)*
	3 groups	*13 singles (base six)*
	4 groups	*3 singles (base six)*

Show 46 base ten + 27 base ten in expanded notation with Color Chip Trading, then symbolically represent this,

e.g.,	*4 blue*	*6 yellow*		*4 tens*	*6 ones*
+	*2 blue*	*7 yellow*	+	*2 tens*	*7 ones*
	6 blue	*13 yellow*		*6 tens*	*13 ones*
	6 blue	*1 blue 3 yellow*		*6 tens*	*1 ten 3 ones*
	7 blue	3 yellow		7 tens	3 ones

Why is expanded notation algorithm better than the standard algorithm?
What are the steps between using manipulatives and the standard algorithm?

Subtraction of Whole Numbers

When do we ***subtract*** in the real world? Give a situation for each of the three models.
Is subtraction always either"take-away" or "comparison" or "missing addend?"
What do we have to know ***before*** we subtract the numbers?

Do students need to know their basic subtraction facts, e.g., 7 - 5?
How many basic subtraction facts are there?
Of the 100 subtraction facts, how many do students really need to know?
Why do we not have a "subtraction chart" for these facts as we do with addition?
Does the conceptual strategy of "missing addend" work for all the subtraction facts?

How does the commutative property for subtraction fit into this idea? Does 7 - 5 equal 5 - 7? Why?

How will we subtract numbers greater than 18 - 9?
Do place value charts and base ten blocks ("Manipulite" Blocks)
Subtract 25 base ten - 14 base ten.
Subtract 41 base ten - 17 base ten. Why is this exercise more difficult for students?
What concept is important? Why is ***regrouping*** the "key?"
Why are 75% of students' add, subtract, multiply, and divide algorithms errors really place value errors?

Show 41 base ten - 17 base ten in expanded notation symbolically,

e.g.,	*4 group 1 single (base ten)*	*3 group 11 single (base ten)*
–	*1 group 7 singles (base ten)*	– *1 group 7 singles (base ten)*
		2 groups 4 singles (base ten)

Why is expanded notation algorithm better than the standard algorithm?
What are the steps between using manipulatives and the standard algorithm?

How does the method of equal addends work,

e.g.,	*4 group 1 single (base ten)*	*4 group 4 single (base ten)*
–	*1 group 7 singles (base ten)*	– *2 group 0 singles (base ten)*
		2 groups 4 singles (base ten)

How is this algorithm alike/different than the expanded notation algorithm?

Multiplication of Whole Numbers

When do we ***multiply*** in the real world? Give a situation for each of the three models.
Is multiplication always either “area model” or “repeated addition” or “combinations”?
What do we have to know ***before*** we multiply the numbers?

Do students need to know their basic multiplication facts, e.g., 5 • 7?
How many basic multiplication facts are there?
Of the 100 multiplication facts, how many do students really need to know?
How does the commutative property for multiplication fit into this idea?
How does the distributive property for multiplication over addition fit into such problems as 8 • 7?
Why is 8 • 7 = (8 • 5) + (8 • 2) = 40 + 16 = 56?
What about other strategies such as doubles, and near-doubles (6 • 7 = [6 • 6] + 6) strategies?
What about near-ten (7 • 9 = [7 • 10] – 7) strategies?

How will we multiply numbers greater than 9 • 9?
Do “area model” and base ten blocks (“Manipulite” Blocks)
How can we show 2 • 3? (rectangular area of 2 width and 3 length)
How can we show 2 • 13 (2 • [1 ten 3 ones])?
How can we show 12 • 13?
- See text and MATH 402 packet diagrams -

What concepts are important? Why is ***regrouping and area*** the “keys?”
Why are 75% of students’ add, subtract, multiply, and divide algorithms errors really place value errors?

Show 43 base ten • 12 base ten in expanded notation symbolically,

e.g.,		*4 tens*	*3 ones*	
X		*1 tens*	*2 ones*	
			6 ones	*= 2 ones X 3 ones*
		8 tens		*= 2 ones X 4 tens*
		3 tens		*= 1 ten X 3 ones*
	4 hundreds			*= 1 ten X 4 tens*
	4 hundreds	*11 tens*	*6 ones*	
	5 hundreds	*1 tens*	*6 ones*	

Why is expanded notation algorithm better than the standard algorithm?
What are the steps between using manipulatives and the standard algorithm?

Division of Whole Numbers

When do we ***divide*** in the real world? Give a situation for each of the two models.
Is division always either"repeated subtraction" or "equal distribution/sharing?"
What do we have to know ***before*** we divide the numbers?

Do students need to know their basic division facts, e.g., 48 ÷ 6?
How many basic division facts are there?
Of the 100 division facts, how many do students really need to know?
Why do we not have a "division chart" for these facts as we do with multiplication?
Does the conceptual strategy of "missing multiplier" work for all the division facts?

How does the commutative property for division fit into this idea? Does 35 ÷ 7 equal 7 ÷ 35? Why?

How will we divide numbers greater than 81 ÷ 9?
Do "repeated subtraction" via the"scaffolding model" with Base Ten Blocks or Colored Chip Trading.

How can we show 405 ÷ 13?

What concepts are important? Why is ***regrouping and area*** the "keys?"
Why are 75% of students' add, subtract, multiply, and divide algorithms errors really place value errors?

How many times can we subtract 13 from 405?

First, estimate: more than 1 time? more than 10 times? more than 100 times?
If it is more than 10 times but fewer than 100 times, it must be a two-digit number.

Is it closer to 10 times or 100 times? Thus, if it is closer to 10 times then we can estimate that a reasonable solution would be greater than 10 times and fewer than 50 times.

	per each	*10 times < estimate < 50 times*
13) 475		
– 130	*10 times*	
345		
– 130	*10 times*	
215		
– 130	*10 times*	
85		
– 13	*1 time*	
72		
– 26	*2 times*	
46		
– 26	*2 times*	
20		
– 13	*1 time*	
7	*36 times total*	*Thus you can subtract 13 from 475 a total of 36 times with 7 remaining*
		This solution is a two-digit number greater than 10 and fewer than 50

Why is repeated subtraction algorithm better than the standard algorithm?
What are the steps between using manipulatives and the standard algorithm?

Introduction to Fractions

When are fractions used in the real world? Give an example for each of the two models.
Are fractions always either “parts of a whole (area model)” or “parts of a total (ratio model)”?
Why do the pieces (parts) in the area model need to be the same size?
Why do the pieces (parts) in the ratio model ***not*** need to be the same size?

What are the three components of a fraction that we need to know?
What is the meaning of the numerator (number)?
What is the meaning of the denominator (place value name)?
What is the meaning of the unit?

What is the importance of the ***unit*** in a fraction?
When can 1/4 be greater than 3/4? Can 1/4 be equivalent to 3/4?
How is the symbolic representation of a fraction (3 fourths)like whole number number and name in place value?
Why should we say “3 fourths” instead of “3 four” or “third 4” or “third fourths” for meaningful use of math language?

As with whole numbers, give ten “nicknames” (equivalent fractions) for the same quantity, e.g., 4 fifths as 8 tenths.
Why does paper folding demonstrate equivalent fractions?
Why should we call them equivalent fractions and not “equal” fractions?

To order fractions, why do we need to have common place value names (denominators) first?
Does this mean that ***equivalence*** (for equivalent fractions) a key concept for students to understand?

Addition & Subtraction: Fractions

When do we add fractions used in the real world? Give an example for each of the two models.
Why are these the same situations as with addition of whole numbers?

When do we subtract fractions used in the real world? Give an example for each of the three models.
Why are these the same situations as with subtraction of whole numbers?

In addition and subtraction of fractions, why do we need to have common place value names (denominators) first, then add or subtract the numbers (numerators)?

Multiplication & Division: Fractions

When do we multiply fractions used in the real world? Give an example for each of the three models.
Why are these the same situations as with multiplication of whole numbers?

When do we divide fractions used in the real world? Give an example for each of the two models.
Why are these the same situations as with division of whole numbers?

What are the two types of fraction multiplication exercises?
In whole number X fraction, why do we do this in the repeated addition model?
In fraction X fraction, why do we do this in the rectangular area model (intersecting areas)?

In fraction division, why do we do this as repeated subtraction?
In fraction division, why do we need to have common place value names (denominators) first, then divide the numbers (numerators)?
Why is “invert and multiply” not logical at the elementary level? Why should this method not be used until algebra?

Do the Condomimnium Problem in groups:

> If 2/3 of the men are married to 3/5 of the women in a condominium complex, what is the fraction of whole condominium is married?

Introduction to Decimals

What is the "proper" name for decimals? Why were they called "decimal fractions"?

When are decimals used in the real world? Give an example for each of the two models.
Are decimals always either "parts of a whole (area model)" or "parts of a total (ratio model)"?

Why is it that a decimal point is not the separator of the whole numbers and the decimals in a decimal representation?
Why isn't the place value names in decimal fractions "balanced" on either side of the decimal point?
What does the decimal point "point" at?
As with fractions, what is the most important component of a decimal fraction?
If the unit place value is the "center/balance" of the decimal fraction, one place value to the right is called? - one place value to the left of the unit's called?

Why should we say "0 and 4 tenths" instead of "point 4" for meaningful use of math language?

As with whole numbers, give ten "nicknames" (equivalent decimals) for the same quantity, e.g., 8 tenths as 80 hundredths.
Why does paper folding and place value trading both demonstrate equivalent decimals?
Why should we call them equivalent decimals and not "equal" decimals?

To order decimals, why do we need to have common place value names (denominators) first?
Does this mean that ***equivalence*** (for equivalent decimal fractions) a key concept for students to understand?

Addition & Subtraction: Decimals

When do we add decimals used in the real world? Give an example for each of the two models.
Why are these the same situations as with addition of whole numbers? - as with addition of fractions?

When do we subtract decimals used in the real world? Give an example for each of the three models.
Why are these the same situations as with subtraction of whole numbers? - as with subtraction of fractions?

In addition and subtraction of decimals, what should we say instead of "line up the decimal points"?
In addition and subtraction of decimals, why do we need to have common place value names (denominators) first, then add or subtract the numbers (numerators)?

Multiplication & Division: Decimals

When do we multiply decimals used in the real world? Give an example for each of the three models.
Why are these the same situations as with multiplication of whole numbers? - as with fractions?

When do we divide decimals used in the real world? Give an example for each of the two models.
Why are these the same situations as with division of whole numbers? - as with fractions?

What are the two types of decimal multiplication exercises?
In whole number X decimal, why do we do this in the repeated addition model?
In decimal X decimal, why do we do this in the rectangular area model (intersecting areas)?

In decimal division, why do we do this as repeated subtraction?
In decimal division, why do we need to have common place value names (denominators) first, then divide the numbers (numerators)?
Why is "invert and multiply" not logical at the elementary level? Why should this method not be used until algebra?

In multiplication of decimals, what should we say instead of "add up the decimal places"?
In multiplication of decimals, why do we need to have place value names (denominators) first for the answer, then multiply the numbers for the numerator of the answer?

In division of decimals, what should we say instead of "move over the decimal places"?
In division of decimals, why do we need to have common place value names (denominators) first, then divide the numbers for the numerator of the answer?
Could we use equivalent fractions for decimal division also, e.g., $0.2 \div 0.04 = 2 \div 0.4 = 20 \div 4 = 200 \div 40 = \ldots$?

Percents, Ratio, Proportions

- Ratio: Part to Whole Comparisons; Part to Part Comparisons
- Proportions: Equivalence of Two Ratios
- Methods of Solving Real-World Proportion Problems: Additive Model; Nice Multiple Model; Unit Ratio Model (for all elementary students); Means-and-Extremes Model (for algebra students)
- Percents: Common Fractions of "Per Hundredths;" Equivalent Relationships among Fraction, Decimals, and Percents
- Methods of Solving Real-World Percentage Problems: Proportion Approach Using Rectangular Area Diagrams and Unit Ratio Model

Measurement, Area, & Volume

- Time as Numberline Clock
- Money as Money Grids
- Measurement in One-Dimension: Length; Perimeter; Circumference; Angles
- Measurement in Two-Dimensional Shapes: Estimation; Nonstandard Units; Need for Standard Units; Area in Square Units; Base Multiplied by Height
- Connections of all Two-Dimensional Areas to Rectangle (Squares, Triangles, Parallelograms, Trapezoids, Pentagons, ... Circles)
- Measurement in Three-Dimensional Shapes: Estimation; Nonstandard Units; Need for Standard Units; Volume in Cubic Units; Base Area Multiplied by Height
- Transformational Geometry: Mira Mathematics for Reflections, Rotations, Translations

Geometry

- Geometry: Describing Shapes and their Relationship to the Real World; Properties; Attributes

- van Hiele Theory of Children's Development in Understanding Geometry:
  Level 0 (Recognition);
  Level 1 (Analysis);
  Level 2 (Relationships);
  Level 3 (Deduction);
  Level 4 (Axiomatics)

-Geometry Vocabulary: Why Specifics Needed
- Development of Language: From Real World to Geometric Definitions and Attributes via Examples and Counter Examples
- Measurement: Number and Place Value Name; Meaning
- Measurement Process of Development: Estimation; Nonstandard Units; Need for Standard Units; English Units of Measure; Metric (Base Ten) Units of Measure (Syste`me International - SI)

Algebra Connections

- Algebraic Areas: Base Ten Block Multiplication; Algebraic Multiplication of Binomials Using Algebra Blocks and Cartesian Coordinate System; Factoring of Binomials Using Algebra Blocks and Cartesian Coordinate System

ALGEBRA IS (ARE): The Different Concepts of Algebra

1. Algebra as generalized arithmetic (patterns):
   5 times 1 = 5;
   5 times 2 = 10;
   5 times 3 = 15; ... 5a

2. Algebra as the study of procedures for problems
   (variables are unknown):
   5X + 3 = 40

3. Algebra as the study of relationships among quantities
   (variables that vary):
   A = LxW
   "graph"
   Find an equation of a straight line ...

4. Algebra as the study of structures:
   Derive sin2x +cos2x = 1
   Justify why sin2x +cos2x = 1

5. Algebra as programming language:
   Let x = x + 1

Write as an equation for:
"There are six times as many students as professors"

Concatenation: 8X means 8 times X,
but 46 does not mean 4 times 6

Mathematical Connections, Assessment & Evaluation of Student Learning

UNDERSTANDING THE ASSESSMENT LANGUAGE IN MATHEMATICS EDUCATION

Assessment is the systematic process of **GATHERING** evidence.
Evaluation is the process of **INTERPRETING** that evidence and making decisions based upon the interpretations.
Thus, IGAP (Illinois Goals Assessment Progress) is an assessment, not an evaluation.
Assessment and evaluation are **processes** not **products.**

In evaluating the quality of the assessment, there are two key components that must be comprehended:
reliability and **validity**.
Reliability is the degree of consistency (dependability) of the assessment and evaluation process.
Validity is the extent to which an assessment actually measures what it is intended to measure.

Reporting is the process of communicating the evaluation.
Accountability is the responsibility of reporting assessment results.
Accountability has a negative connotation in education, one that only implies punishment.
However, accountability can provide a valuable opportunity for learning.

TYPES OF ASSESSMENTS

Formative Assessment is an ongoing process to plan and modify instruction.
Summative Assessment is carried out at the end of a time period.
Diagnostic Assessment is an in-depth process to identity special needs or areas where a student has particular difficulty.
Alternative Assessment is a term invented to contrast with "traditional" forms of assessment.
Alternative assessment implies that the learner **generates** ideas/concepts/proofs rather than **responds** to questions.
Performance Assessment is a process that requires action, a performance of some kind, on the part of the learner.
Authentic Assessment is a application assessment. The learner is to reproduce the application or practical aspects of the mathematics to a real-world situation.
Problem Solving questions are challenges the learner does not have a direct algorithm for the solution. Looking for number patterns is a problem solving challenge, but not a performance assessment nor an authentic assessment.

PROGRESS, GROWTH, NORMATIVE COMPARISONS

Academic Growth implies a change for the better in the learner, but does not imply that there is a defined objective.
Academic Progress also implies growth, but it is a measure backwards from a specific educational goal.
Academic growth is a comparison with one's previous self; academic progress is a comparison towards an established goal.
In some instances a more useful tool for communicating about mathematical achievement may be normative comparisons. While any measurement is a comparison, local **Normative Comparisons** present information in context of other students' rates of progress.

REPORTING PROGRESS

One especially difficulty problem with any type of assessment is communicating progress to parents.
Parents feel that they understand the "A, B, C, D, F" grading from their own experiences in school.
There are two large myths parents are unaware with the traditional "A, B, C, D, F" grading scheme.
First, there is the general perception that "A, B, C, D, F" grading is consistent, i.e., that it has reliability.
The second myth is the view that "A, B, C, D, F" grading is objective.

Prevalent faith in our system that academic competition between students is the best method for reporting assessment
The upper echelon of students may experience motivation from this method, research shows (NCTM, 1995) lower-level students develop self-defeating expectations.

A better method in mathematics is to move away from a practice of comparing students' performance with other students and towards a practice of comparing students' attainment with performance criteria in a scoring rubric.

One type of assessment may be necessary but generally never is sufficient.
A balance of several types of assessments such as performance events with best-piece mathematics portfolios and traditional assessments, may be best.

Good assessment should be a part of instruction, not a separate activity.
Good assessment emphasizes the role of the student as an active mathematician and the teacher as a guide.
Good assessment promotes a vision of mathematics that includes but goes beyond correct answers.

ASSESSMENT SCORING RUBRIC

- The lowest or **novice/beginning level** exhibits work that has inappropriate strategies, no justification of processes, and serious errors in the mathematics.
- The next level, **apprentice/developing**, shows work that has vague explanations of the process(es) used, some mathematical errors, and inappropriate problem-solving strategies.
- The third level, **proficient/competent**, displays work that is well developed, clear communications, and process(es) based upon logical reasoning.
- The highest level, **distinguished/expanding**, goes well beyond the original problem. Not only are the strategies most appropriate, alternate strategies are discussed and reasons for their non-use are justified. The process(es) used is described precisely, concisely, and in detail.

Review & Reflection

National Council of Teachers of Mathematics (NCTM) FIVE GENERAL GOALS FOR ALL STUDENTS (1989)

All Students Should: Learn to Value Mathematics
Become Confident in Their Ability
Become Math Problem Solvers
Learn to Communicate Math
Learn to Reason Mathematically

NCTM CURRICULUM AND EVALUATION STANDARDS FOR K-4 SCHOOL MATHEMATICS (1989)

1. Mathematics as Problem Solving
2. Mathematics as Communication
3. Mathematics as Reasoning
4. Mathematical Connections
5. Estimation
6. Number Sense and Numeration
7. Concepts of Whole Number Operations
8. Whole Number Computations
9. Geometry and Spatial Sense
10. Measurement
11. Statistics and Probability
12. Fractions and Decimals
13. Patterns and Relationships

NCTM CURRICULUM AND EVALUATION STANDARDS FOR 5-8 SCHOOL MATHEMATICS (1989)

1. Mathematics as Problem Solving
2. Mathematics as Communication
3. Mathematics as Reasoning
4. Mathematical Connections
5. Number and Number Relationships
6. Number Systems and Number Theory
7. Computation and Estimation
8. Patterns and Functions
9. Algebra
10. Statistics
11. Probability
12. Geometry
13. Measurement

NCTM PROFESSIONAL STANDARDS FOR TEACHING MATHEMATICS (1991)

1. Worthwhile Mathematical Tasks
2. The Teacher's Role in Discourse
3. Students' Role in Discourse
4. Tools for Enhancing Discourse
5. Learning Environment
6. Analysis of Teaching and Learning

NCTM ASSESSMENT STANDARDS FOR SCHOOL MATHEMATICS (1995)

1. The Mathematics Standard — Assessment should reflect the mathematics that students need to know and be able to do.
2. The Learning Standard — Assessment should enhance mathematics learning.
3. The Equity Standard — Assessment should promote equity.
4. The Openness Standard — Assessment should be an open process.
5. The Inferences Standard — Assessment should promote valid inferences about mathematics learning.
6. The Coherence Standard — Assessment should be a coherent process.

Use of the Assessment Standards for Different Purposes

1. Monitoring Students' Progress
2. Making Instructional Decisions
3. Evaluating Students' Achievement
4. Evaluating Programs