

ROLLING RECTANGLES ACTIVITY

MATHEMATICAL CONTENT: Measurement (Volume vs. Surface Area)

TIME: 20 minutes

MATHEMATICAL BACKGROUND: Students should have been introduced to the concept of volume and to the “basic” formula for calculating the volume of a solid (base area times height). Students also need to know about the concept of area (lateral surface) and the formula for the area of a circle (πr^2)

MATERIALS: Each group should have: one sheet of heavy typing paper (8.5” by 11”), one pair of scissors, masking or transparent tape, one tray, one large bag of beans or other filler such as rice (about one quart).

GROUPING OF STUDENTS: Pairs; can also do cooperative groups of four students subdivided into pairs.

PREPARATION: Divide students into groups and have one student from each group collect the materials for that group.

INSTRUCTIONS FOR TEACHERS:

1. Introduction

Have each group cut the piece of paper in half to make two equal size rectangles. Some groups will (should) have different-sized rectangles than others (if not, cut a piece of paper yourself to produce one). Ask students to explain how it possible to have two different rectangles that are half the size of the original rectangle. You may want to physically compare the two-different-sized rectangles to show that they do have the same area.

2. Ask each pair of students to carefully roll one piece of paper into the largest cylinder (a tube without a top or bottom) they can. When finished, there should be little or no overlap of the paper. Have the students securely tape this cylinder.

3. Next, ask each pair of students to carefully roll the other piece of paper into the largest cylinder (a tube without a top or bottom) that is different from the first cylinder. They should then securely tape this cylinder. Each pair of students should now have different two cylinders - one tall and skinny, the other short and dumpy (fat) - that have the same lateral surface area.

4. Ask the students as a class how the two cylinders are alike and how they are different.

5. Ask each student to predict which of the two cylinders would hold the most beans (or will they hold the same amount). Then ask the students in each group to discuss their ideas and predictions and make a group prediction. Tabulate the group predictions and ask 2-4 of the groups to explain the reasons for their predictions.

6. To save instructional time, have the students place the tall/skinny cylinder inside the short/dumpy cylinder and then fill the tall/skinny cylinder full of beans. On your command, have all the groups CAREFULLY pull out the tall/skinny cylinder, allowing the beans to flow into the short/dumpy cylinder.

7. Closure

- a. Ask students if their predictions were correct (refer back to the tabulated group predictions). Were they surprised by the results? What did they learn?
- b. Give groups a few minutes to discuss why it is that the short/dumpy cylinder has a much greater volume (holds more beans) than the tall/skinny cylinder. If the students do not provide the

following explanation (or something close to it), lead them through the following discussion on why the radius of the circular base of the cylinder has a greater effect on its volume than the height:

- One difference between the two cylinders was that the diameter, and thus the radius, of short/dumpy was greater than that of tall/skinny.
 - The volume of each cylinder is found by multiplying its height by its base area.
 - The base area of each cylinder is π times the radius times the radius since the base of each cylinder is a circle, meaning the volume of each cylinder is π times the **radius** times the **radius** times the **height**.
 - Thus, a small change in the radius of the base has a greater effect on the volume of the cylinder than a similar change in the height since the radius is multiplied *twice* and the height only once.
 - c. Ask students if they can think of any applications of this knowledge. For example, how would they make soup cans to hold the most soup (or tuna) using the least materials for the sides of the cans? Before asking this question, you may wish to show students different-sized (soup) cans.
8. Homework
- a. Have students find and record the volume and surface areas of various cans of grocery items (peas, beans, tuna). Students will need to measure accurately the height, diameter, and circumference.
 - b. Have the students identify a grocery product that could be made with less lateral surface area. Have students work out the dimensions and surface areas of a new can that would have the same volume. (This could lead into an inter-discipline project where students write to the engineers of the company, suggesting a new design.)

QUESTIONS TO BE ADDRESSED (INCLUDING REFLECTIVE QUESTIONS)

During the activity:

- How is it possible to cut a piece of paper and produce two different rectangles that are both half the size of the original piece of paper?
Extension: How many different ways can you cut a piece of paper in half (single cut) to make two equal size rectangles?
- How are the two cylinders you created alike? How they are different?
- Which of the two cylinders, tall/skinny or short/dumpy, will hold the most beans or will they hold the same amount? Justify your response.

After the activity (reflection):

- Were your predictions correct? Why or why not? What did you learn from this?
- Why is it that the short/dumpy cylinder has a much greater volume (holds more beans) than the tall/skinny cylinder? Explain.
- In what (real-life) situations might you make use of this knowledge about the relationship between the radius and height of a cylinder and its volume?

SOURCE: *The Good Time Math Event Book* by Marilyn Burns, Creative Publications, 1977; Adapted by Dr. Donald Porzio from a problem-solving presentation write-up by Dr. Alan Zollman, Northern Illinois University.