FALL 2003

MATH 434
EXAM 2

NAME__________________

1. Let T be an $n \times n$ symmetric positive definite tridiagonal matrix.

   (a) Write an algorithm to compute $T = LU$ where L is a lower unit biodiagonal matrix.

   (b) Determine the flop counts.

   (c) Let $T = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix}$, $b = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$. Using algorithm in (a) solve $Tx = b$.

2. (a) Define condition number of a matrix.

(b) Let A be a nonsingular matrix. Then prove that

(i) $Cond(A) \geq 1$

(ii) $Cond_2(A^T A) = (Cond_2(A))^2$

(c) Let $A = \begin{pmatrix} 1 & c \\ c & 1 \end{pmatrix}$, $c \neq 1$. Calculate $Cond(A)$. When does A become ill-conditioned?

3. (a) Define a convergent matrix.

(b) Suppose we want to solve $Ax = b$ using following iterative scheme

$$x^{(k+1)} = Bx^{(k)} + d \tag{1}$$

(i) What is the necessary and sufficient condition for the iteration (1) to converge to an exact solution starting from an arbitrary initial guess $x^{(1)}$.

(ii) Write down the matrices B for both Jacobi and Gauss Seidal methods.

(iii) Show that if A is strictly diagonally dominant that $||B_J||_\infty < 1$.

(c) Do just two iterations of the Gauss-Seidal method for the System

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix} \text{ with } x^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

(i) Does the Gauss Seidal method converge for the above system for any intial vector? Justify your answer.

(ii) Compute

a. $\dfrac{||x^{(2)} - x^{(1)}||_\infty}{||x^{(1)}||_\infty}$

b. What is your conclusion?