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MATH 434
Fall 2004
Exam 1

1. **25 pts.**

   (a) Define Elementary matrix.

   (b) Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. Find two elementary matrices E_1 and E_2 such that $E_2 E_1 A$ is an upper triangular matrix.

   (c) Express $A = LU$ using (b) where L is a unit triangular matrix.

   (d) Express inverse of A in a factor form using (c). Also find det A.

2. **25 pts.** Let A be a tridiagonal matrix of the following form:

$$A = \begin{pmatrix} a_1 & b_1 & 0 & \dots & 0 & 0 \\ b_1 & a_2 & b_2 & \dots & 0 & \\ 0 & b_2 & a_3 & b_3 & 0 & \\ & & & \ddots & \ddots & \\ & & & b_{n-2} & a_{n-1} & b_{n-1} \\ 0 & & & 0 & b_{n-1} & a_n \end{pmatrix}$$

   (a) Write an algorithm to compute

$$A = LU$$

   where L is an unit lower triangular matrix. (when it exists)

   (b) Find the total flop count.

3. **20 pts.**

   (a) Suppose $fl(x)$ denote the floating point representation of a real number x using t digit rounding.
   Show that $\dfrac{|fl(x) - x|}{|x|} \leq .5 \times 10^{1-t}$.
   or

   (a) Show that the floating point computations of the sum and quotient of two numbers are forward and backward stable.

4. **30 pts.**

   (a) Define $||A||_2$ and $||A||_F$

   (b) Show that $||QAP||_F = ||A||_F$ and $||QA||_2 = ||A||_2$
   where Q, P are orthogonal matrices.

   (c) Show that if P, Q are orthogonal matrices then (PQ) is also an orthogonal matrix.