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MATH 434
Fall 2004
Exam 2

1a. **(5 pts.)** Define the condition number of a matrix.

1b. **(10 pts.)** Suppose that in the problem $Ax = b$, the vector b is subjected to purbation but not the matrix A. Let A be a nonsingular matrix, then prove the inequality.

$$\frac{||\delta x||}{||x||} \leq \text{ Cond } (A) \frac{||\delta b||}{||b||}$$

where $Ax = b$ and $A(x + \delta x) = b + \delta b$.

1c. **(10 pts.)** Let $A = \begin{pmatrix} .01 & .011 \\ .011 & .1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 113.77 & -12.51 \\ -12.51 & 11.3766 \end{pmatrix}$.

Suppose we want to solve $Ax = \begin{pmatrix} 1 \\ .01 \end{pmatrix}$ and let the x_c (computed solution) be $\begin{pmatrix} 113 \\ -12 \end{pmatrix}$. Then estimate the relative error bound $\frac{||\delta x||}{||x||}$ using $1(b)$.

2a. **(10 pts.)** Let x be an n-vector. Write an algorithm to compute a householder matrix

$H = I - 2\frac{uu^T}{u^T u}$, such that Hx has zeros in the 2nd position through n^{th}, ie $Hx = \begin{pmatrix} * \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

2b. **(10 pts.)** Given $x = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 1 \end{pmatrix}$, find H such that $Hx = \begin{pmatrix} * \\ 0 \\ 0 \\ 0 \end{pmatrix}$

3a. **(10 pts.)** Develop an algorithm to solve a symmetric positive definite Tri-diagonal system using Cholesky Factorization.

3b. **(10 pts.)** Let $T = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Solve the system $Tx = b$ using algorithm in $3(a)$

4a. **(5 pts)** Does LU factorization of A using G.E. without pivoting always exist? Give reasons for your answer. What are the Numerical difficulties?

4b. **(10 pts.)** Let $A = \begin{pmatrix} .001 & 1 \\ 2 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 2.001 \\ 3 \end{pmatrix}$

Solve $Ax = b$ using (GE with partial pivoting) with explicit factorization of A.

5a. **(20 pts.)** Define a convergent matrix

5b. Find the Jacobi matrix B_J for a given system $Ax = b$

5c. Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix.

Write down the inequalities that will guarantee the convergence of Jacobi iteration with any arbitrary guess $x^{(1)}$.

5d. Let $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ 5 \\ 4 \end{pmatrix}$. Using Jabobi iteration find the solution x for $Ax = b$ with $x^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. (Just do 2 iterations)