Exercises for Math 480 Week #9

1: Using any method you want, find two rational numbers \( p/q \) satisfying

\[ |\sqrt{3} - (p/q)| < \frac{1}{\sqrt{5}q^2}. \]

2. Following the proof of Liouville’s Theorem, find a number \( C > 0 \) so that

\[ |\sqrt{3} - (p/q)| \geq \frac{C}{q^3} \]

for any rational number \( p/q \).

3. Using Newton’s method on \( f(X) = X^2 - 11 \) with an initial “guess” of 3, compute the next three rational approximations to \( \sqrt{11} \). Do any of these give you a solution in integers to \( X^2 - 11Y^2 = \pm 1 \), whence a unit in \( \mathbb{Z}[\sqrt{11}] \)?

4. Following the proof of Dirichlet’s Theorem, find three rational approximations \( p/q \) satisfying

\[ |e - p/q| < 1/q^2. \]