Math 680 Fall 2017
Third Homework Assignment

Exercise 7. Under the assumption that there are no primes \( p \) with \( n < p \leq 2n \), we showed that

\[
\left( \frac{2n}{3} - 1 - \sqrt{2n} \right) \log 2 < \left( \sqrt{2n} + \frac{1}{2} \right) \log n.
\]

Find a lower bound \( n_0 \) (not necessarily optimal) such that the above inequality is false for \( n \geq n_0 \).

Exercise 8. Check, via a table of primes, that for all \( 1 \leq n < n_0 \) (where \( n_0 \) is as above) there is a prime \( p \) with \( n < p \leq 2n \). Conclude that \( \pi(2n) - \pi(n) > 0 \) for all \( n \geq 1 \).

Exercise 9. Show that the alternating series

\[
F(\sigma) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\sigma}}
\]

converges for all \( \sigma > 0 \) and that

\[
F(s) = \frac{1}{1 - 2^{1-s}} = \zeta(s)
\]

for all complex \( s = \sigma + it \) with \( \sigma > 1 \).