Math 680 Fall 2017

Third Homework Assignment

Exercise 7. Under the assumption that there are no primes p with $n < p \leq 2n$, we showed that

$$\left(\frac{2n}{3} - 1 - \sqrt{2n}\right) \log 2 < \left(\sqrt{2n} + \frac{1}{2}\right) \log n.$$

Find a lower bound n_0 (not necessarily optimal) such that the above inequality is false for $n \geq n_0$.

Exercise 8. Check, via a table of primes, that for all $1 \leq n < n_0$ (where n_0 is as above) there is a prime p with $n < p \leq 2n$. Conclude that $\pi(2n) - \pi(n) > 0$ for all $n \geq 1$.

Exercise 9. Show that the alternating series

$$F(\sigma) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^\sigma}$$

converges for all $\sigma > 0$ and that

$$\frac{F(s)}{1 - 2^{1-s}} = \zeta(s)$$

for all complex $s = \sigma + it$ with $\sigma > 1$.