Exercise 10. We showed that for all \( s = \sigma + it \) with \( \sigma > 1 \),
\[
\zeta(s) = \prod_p (1 - p^{-s})^{-1}.
\]
Suppose \( f \) is a multiplicative function and \( \sum_{n=1}^{\infty} |f(n)|n^{-\sigma_0} \) converges. Show that
\[
\sum_{n=1}^{\infty} \frac{f(n)}{n^s} = \prod_p \left( 1 + f(p)p^{-s} + f(p^2)p^{-2s} + \cdots \right),
\]
and if \( f \) is totally multiplicative
\[
\sum_{n=1}^{\infty} \frac{f(n)}{n^s} = \prod_p \left( 1 - f(p)p^{-s} \right)^{-1}
\]
for all \( s = \sigma + it \) with \( \sigma \geq \sigma_0 \).

Exercise 11. Write \( \sum_{n=1}^{\infty} \phi(n)n^{-s} \), \( \sum_{n=1}^{\infty} \sigma(n)n^{-s} \) and \( \sum_{n=1}^{\infty} |\mu(n)||n^{-s} \) in terms of the zeta function as we did in class with \( \sum_{n=1}^{\infty} \mu(n)n^{-s} \) and \( \sum_{n=1}^{\infty} \lambda(n)n^{-s} \), i.e., use the Euler product.

Exercise 12. Write \( \zeta'(s) \) as a Dirichlet series. What is the abscissa of convergence?

Exercise 13. Show that \( \lim_{\sigma \to \infty} \zeta'(\sigma) = 0 \) and that \( \frac{1}{\zeta'(s)} \) can not be written as a convergent Dirichlet series.