Math 680 Fall 2017

Fourth Homework Assignment

Exercise 10. We showed that for all $s = \sigma + it$ with $\sigma > 1$,

$$\zeta(s) = \prod_p \left(1 - p^{-s}\right)^{-1}.$$

Suppose f is a multiplicative function and $\sum_{n=1}^{\infty} |f(n)| n^{-\sigma_0}$ converges. Show that

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^s} = \prod_p \left(1 + f(p)p^{-s} + f(p^2)p^{-2s} + \cdots\right),$$

and if f is totally multiplicative

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^s} = \prod_p \left(1 - f(p)p^{-s}\right)^{-1}$$

for all $s = \sigma + it$ with $\sigma \geq \sigma_0$.

Exercise 11. Write $\sum_{n=1}^{\infty} \phi(n) n^{-s}$, $\sum_{n=1}^{\infty} \sigma(n) n^{-s}$ and $\sum_{n=1}^{\infty} |\mu(n)| n^{-s}$ in terms of the zeta function as we did in class with $\sum_{n=1}^{\infty} \mu(n) n^{-s}$ and $\sum_{n=1}^{\infty} \lambda(n) n^{-s}$, i.e., use the Euler product.

Exercise 12. Write $\zeta'(s)$ as a Dirichlet series. What is the abscissa of convergence?

Exercise 13. Show that $\lim_{\sigma \to \infty} \zeta'(\sigma) = 0$ and that $\frac{1}{\zeta'(s)}$ can not be written as a convergent Dirichlet series.