Exercise 14a. Show that 
\[ \Gamma(s) = \lim_{n \to \infty} \frac{n^s n!}{s(s + 1)(s + 2) \cdots (s + n)} \]
for all \( s \neq 0, -1, -2, \ldots \)

b) Show that \( \Gamma(s + 1) = s\Gamma(s) \) for all \( s \neq 0, -1, -2, \ldots \)

c) Show that \( \Gamma(1) = 1 \) and, more generally, \( \Gamma(n + 1) = n! \) for all non-negative integers \( n \).

Exercise 15. Show that
\[ \lim_{N \to \infty} \prod_{n=1}^{N} \frac{(2n)^2}{(2n - 1)(2n + 1)} = \frac{\pi}{2}, \]
and thus
\[ \lim_{n \to \infty} \frac{2^{2n}(n!)^2}{(2n + 1)!} \sqrt{2n + 1} = \sqrt{\frac{\pi}{2}}. \]

Hint: we proved an infinite product representation for the sine function, and you know the value of \( \sin(\pi/2) \).

Exercise 16a. Show that
\[ \Gamma(s)\Gamma(1 - s) = \frac{\pi}{\sin(\pi s)} \]
for all \( s \not\in \mathbb{Z} \). Hint: Use exercise 14a.

b) Show that
\[ \Gamma(s)\Gamma(s + (1/2)) = \sqrt{\pi}2^{1-2s}\Gamma(2s). \]

For what values of \( s \) is this valid? Hint: use exercises 14a and 15.

Exercise 17. Show that
\[ \zeta(s) = 2(2\pi)^{s-1} \sin(\pi s/2)\Gamma(1 - s)\zeta(1 - s). \]

Hint: use the functional equation we proved in class together with the exercises above.