

Math 680 Fall 2017

Eighth Homework Assignment

Exercise 23. For all $s = \sigma + it$ with $0 < \sigma < 1$, show that

$$\int_0^\infty u^{s-1} e^{-iu} du = e^{-i\pi s/2} \Gamma(s).$$

Hint: integrate $u^{s-1} e^{-u}$ along the quarter circle contour of radius R in the first quadrant and let $R \rightarrow \infty$.

Exercise 24. Using exercise 23, show that

$$\int_0^\infty u^{\sigma-1} \sin u du = \Gamma(\sigma) \sin(\pi\sigma/2)$$

for all $0 < \sigma < 1$. Then let $\sigma \rightarrow 0^+$ to find $\int_0^\infty (\sin u/u) du$.

Exercise 25: Show that

$$\left| \int_B^\infty \frac{\sin u}{u} du \right| \ll \min\{1, 1/B\}$$

for all $B > 0$, where the implicit constant is absolute. Hint: integrate by parts for the case where $B \geq 1$.