

NO CALCULATORS!

Show all necessary work. Be neat, clear, and brief.

1. (40 points) Find the derivative of each of the following functions. You do not need to simplify your answers.

(a) $f(x) = \frac{7x^5 + 2x + 6}{2x - 4}$

(b) $f(x) = \ln(3x^4 - 7x^3)$

(c) $f(x) = 5e^{\sqrt{x}}$

(d) $f(x) = (x^7 - 1)^4(3x + 5)^3$

(e) $f(x) = x^2 \ln x$

2. (32 points) Find the following limits or state that no limit exists.

$$(a) \lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x^2 - 8x + 16} =$$

$$(b) \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3} =$$

$$(c) \lim_{x \rightarrow \infty} \frac{5x^4 + 7x^3 - 6}{3x^4 + 2x + 1} =$$

$$(d) \lim_{x \rightarrow \infty} \frac{6x^2 + 2}{5x^3 + 3x} =$$

3. (10 points) In order to sell x units of a certain product, the price per unit must be set at $p = 150 - 0.5x$ dollars per unit. If the cost function is $C(x) = 4000 + 0.25x^2$, determine the level of production that will maximize the profit.

4. (10 points) Find the equation of the function $P(t)$ satisfying the following two conditions.

$$P'(t) = -.05P(t)$$

$$P(0) = 1000$$

5. (14 points) The population of a colony of bacteria doubles every 9 hours.

(a) Find the growth constant for the population.

(b) How long will it take an initial population of 500 bacteria to grow to 5,000?

6. (10 points) Find the absolute maximum and absolute minimum of the function $f(x) = x^4 - 4x^2 + 5$ on the interval $[-1, 1]$.

7. (10 points) Find the equation of the line tangent to the curve $y = \sqrt{x}$ at the point $(9, 3)$.

8. (10 points) Suppose that the function f satisfies the two conditions

$$f'(x) = 3x^2 + 6x - 7 \text{ and } f(1) = 2.$$

Find $f(x)$.

9. (24 points) Compute the following integrals.

(a) $\int_1^4 (3x^2 - 2) dx =$

(b) $\int \left(\frac{3}{x} - \frac{4}{x^2} \right) dx =$

(c) $\int (\sqrt{x} + e^{-3x}) dx =$

10. (10 points) Find the area of the region bounded by the curves $y = 4x - x^2$ and $y = x$.

11. (18 points) Let $f(x) = x^3 - 9x^2 + 15x$.

a) Find all relative extrema of f (BOTH COORDINATES). Determine whether each extremum is a relative maximum or a relative minimum.

b) Find all inflection points of f (BOTH COORDINATES).

c) Using your knowledge of calculus, sketch a graph of f that includes all the points found in parts a) and b).

12. (12 points) Using the limit definition of the derivative (not the power rule), find $f'(x)$

if $f(x) = \frac{1}{2x+1}$.