Department of Mathematical Sciences

QUALIFYING EXAMINATION FOR THE PH.D. IN MATHEMATICAL SCIENCES
COMPREHENSIVE EXAMINATION FOR THE M.S. IN MATHEMATICAL SCIENCES

These written examinations are given three times each year, just before the start of each semester (mid-January, mid-June, and mid-August). The following individual exams are available: **A** (Algebra), **B** (Real and Complex Analysis), **C** (Functional Analysis and Topology), **D** (Differential Equations), **E** (Numerical Mathematics), **F** (Mathematics Education: Foundations and Research), **G** (Mathematics Education: Theories of Learning and Teaching), **H** (Statistics: Probability and Inference), **I** (Statistics: Linear Models and Bayesian Statistics), **J** (Statistics: Advanced Statistical Methods and Statistical Consulting), **ME** (Mathematics Education: Education, Master's level), and **MM** (Mathematics Education: Mathematics, Master's level). All exams are three hours in length (two hours for Master's level) except for Exams **F** and **J** which are take-home exams.

Qualifying Examination

The following guidelines apply for doctoral students and also for master's students who are considering the possibility of going on for a doctorate. The Qualifying Examination consists of three individual exams from above, chosen as follows depending on the student's area of focus.

Mathematics: Two exams chosen from the set {A, B, C}; and any other exam in the list A-I.

Mathematics Education: One exam chosen from the set {A, B, C}; and exams F and G.

Statistics: Exams H, I and J.

Successful completion of the Qualifying Examination is passing all three of these individual exams.
A doctoral student will be expected to attempt at least two of the individual exams within two years of entering the program, and is encouraged to attempt three. On the first attempt, at least two of the individual exams must be taken. On the basis of the outcome, the Graduate Studies Committee will recommend that the student either continue in the doctoral program, complete a master's degree and leave the program, or simply leave the program.

The student may petition the committee to retake one or more parts of the qualifying exam. The committee may, at its discretion, allow a maximum of two attempts to pass any individual exam and a maximum of three attempts to pass the entire Qualifying Examination. The student must take at least two individual exams at any given time, unless only one remains to be passed.

Master's students who are considering the possibility of continuing for a doctorate should note the following:

- Successful performance on Exams A and B is considered to meet the requirement for a Master's Comprehensive Exam in Pure Mathematics.

- Successful performance on Exams B and D is considered to meet the requirement for a Master's Comprehensive Exam in Applied Mathematics.

- Successful performance of Exams B and E is considered to meet the requirement for a Master's Comprehensive Exam in Computational Mathematics.

• Successful performance on either Exam A or Exam B is considered to meet the mathematics requirement portion for a Master's Comprehensive Exam in Mathematics Education.

• Successful performance on Exam F or Exam G is considered to meet the mathematics education requirement portion for a Master's Comprehensive Exam in Mathematics Education.

Students in Statistics should consult the Director of the Division of Statistics for advice as to how Exams H and I may be applied to the Master's Comprehensive Exam in Applied Probability and Statistics.

Comprehensive Examination

Students who are only pursuing a terminal master's degree will proceed as follows. Two appropriate parts of the examination must be passed. On the first attempt both parts must be taken at the same session. In the event that the examination is not passed on the first attempt, it may be repeated once. If the candidate passes one part of the examination on the first attempt, and if the examination is repeated within one academic year, then it is only necessary to repeat that part of the examination which was previously failed. In all other circumstances in which the student is taking the examination for the second time, both parts must be repeated.

An exception to this rule is that a master's student in Mathematics Education may, with departmental approval, take the mathematics portion of the Comprehensive Examination after completing the mathematics course requirement of their program. If the student passes the mathematics part of the examination, then it is not required that the student pass the education part within one year. If the student fails the mathematics part on the first attempt, then such a student must take both parts of the examination at the time of the second attempt and pass the mathematics part. If needed, the student may repeat the education part of the examination, provided this is done within one year of the previous attempt.

M.S. in Pure Mathematics: The two-hour version of Exam A and Exam B.

M.S. in Applied Mathematics: The two-hour version of Exam B and Exam D.

M.S. in Mathematics Education: The exam will have a mathematics portion (MM) and an education portion (ME). The mathematics portion (MM) is one of the following choices: the two-hour version of Exam A, the two-hour version of Exam B, a two-hour exam over statistical inference, or a two-hour general examination involving four different mathematics courses the student has taken for graduate credit. The education portion (ME) is a three-hour exam covering Math 610 and at least two courses selected from Math 611, 612, 613, 614, 615, and 617, depending on what the student has taken. Any student in this program should consult with the Director of Graduate Studies to determine the exact nature of their examination.

M.S. in Computational Mathematics: The two-hour version of Exam B and Exam E.

M.S. in Applied Probability and Statistics: Students in Statistics should consult the Director of the Division of Statistics for advice as to the exact nature of their examination.

Syllabi for the above exams are below. A student taking the two-hour version of Exam A will be responsible only for Part I of the syllabus. Approximately 1/4 of this two-hour exam is devoted to linear algebra. A student taking the two-hour version of Exam B will be tested fully over advanced calculus, but will have a considerable choice of questions from real and complex analysis, and may avoid questions from one of these two areas entirely if desired. A student taking Exam E will be tested over Numerical Analysis and either Numerical Linear Algebra or Numerical Differential Equations (the student must make this choice in advance).

SYLLABUS FOR EXAMINATION A

ALGEBRA

PART I

Groups: Groups, subgroups, normal subgroups, homomorphism theorems, Sylow theorems, structure theorem for finite abelian groups, Jordan-Hölder theorem, solvable groups.

Rings: Rings, ideals, homomorphism, field of fractions of an integral domain.

Fields and Galois theory: characteristics, prime fields, algebraic and transcendental extensions, separability, perfect fields, normality, splitting fields, Galois group, fundamental theorem of Galois theory, solvability by radicals, structure of finite fields.

Linear algebra: Linear independence, basis, dimension, direct sums, linear transformations and their matrix representations, linear functions, dual spaces, determinants, rank, eigenvalue and eigenvector, minimal and characteristics polynomials, canonical forms.

Part II

Rings and Modules: Modules, simplicity, semisimplicity, chain conditions, tensor products, Jacobson radical, density theorem, Wedderburn-Artin theorem, finitely generated modules over a principal ideal domain, canonical forms. Unique factorization, Euclidean domains, principal ideal domains, polynomial rings, maximal, prime, and primary ideals, Noetherian rings, Hilbert basis theorem, Lasker-Noether decomposition, integral elements, fractional ideals, Dedekind domains.

Primary References:

Jacobson, Basic Algebra I, II
Hungerford, Algebra
Issacs, Algebra: A Graduate Course
Rotman, The Theory of Groups
Garling, A Course in Galois Theory
Friedberg, Insel, and Spence, Linear Algebra
Hoffman and Kunze, Linear Algebra
Clark, Elements of Abstract Algebra

Secondary References:

Beachy and Blair, Abstract Algebra with a Concrete Introduction
Fraleigh, A First Course in Abstract Algebra
Herstein, Topics in Algebra
Anton, Elementary Linear Algebra

SYLLABUS FOR EXAMINATION B

REAL AND COMPLEX ANALYSIS

Advanced Calculus: Axioms for the real numbers and cardinality of sets. Sequences in one and several real variables. Topology of the line and Euclidean n-space. Limits, continuity, and differentiation of functions in one and several real variables. Integration in one and several real variables. Sequences and series of functions. Elementary functions of real variables and their properties.

Real Analysis: Lebesgue measure and the Lebesgue integral on the line. Borel sets, sigma algebras, outer measure, measurable sets and functions, Egoroff's theorem, Lusin's theorem, Fatou's lemma, Monotone Convergence theorem, Lebesgue convergence theorem, convergence in measure, differentiation of monotone functions and of an integral, absolute continuity.

Complex Analysis: Construction of complex numbers from the reals. Topology of the complex plane. Limits, continuity, and differentiation of functions of one complex variable. Analytic functions, harmonic functions, and the Cauchy-Riemann equations. Elementary functions and their properties. Conformal mapping and linear fractional transformations. Power series. Complex integration and the Cauchy theorems. Local properties of analytic functions, including the open mapping theorem. Residue theory and applications.

References:

Ahlfors, Complex Analysis
Apostol, Mathematical Analysis
Derrick, Complex Analysis and Applications
Fulks, Advanced Calculus
Gaughan, Introduction to Analysis
Kaplan, Advanced Calculus
Pennisi, Elements of Complex Variables
Royden, Real Analysis

SYLLABUS FOR EXAMINATION C

FUNCTIONAL ANALYSIS AND TOPOLOGY

Functional Analysis: Banach spaces, products and quotients of normed linear spaces, dual spaces, Hahn-Banach Theorem, Stone-Weierstrass Theorem, Ascoli's Theorem, second dual space, weak and weak-star topologies, Alaoglu's Theorem, open mapping and closed graph theorems, projections, uniform boundedness, extreme points, Krein-Milman Theorem; inner product and Hilbert spaces, Bessel's inequality, Parseval's relation, adjoint operators, self-adjoint and normal operators, unitary operators; abstract measure theory, Hahn decomposition, Jordan decomposition, Radon-Nikodym Theorem, Riesz Representation Theorem for the dual of $C(K)$.

Topology: General topology (topological spaces, bases, products, subspaces, quotients, continuous maps); metric spaces (continuity, convergence, completion, Baire Category Theorem); compactness properties (compactness, local compactness, compactifications, compactness in metric spaces, Heine-Borel Theorem); covering properties (Lindelof property, paracompactness); separation and countability axioms; Urysohn's Lemma and the Tietze Extension Theorem; Tychonoff Theorem; connectivity (connectedness, path-connectedness, components); homotopy theory (homotopic maps, contractible spaces, deformation retracts); fundamental groups (functorial properties, calculations for euclidean spaces, spheres, relationship to covering spaces).

References:

Croom, Principles of Topology
Dugundji, Topology
Goffman and Pedrick, First Course in Functional Analysis
Hewitt and Stromberg, Real and Abstract Analysis
Massey, Algebraic Topology: An Introduction
Munkres, Topology, A First Course
Pedersen, Analysis Now
Royden, Real Analysis
Rudin, Real and Complex Analysis
Simmons, Introduction to Topology and Modern Analysis
Singer and Thorpe, Lecture Notes on Elementary Geometry and Topology
Willard, General Topology

SYLLABUS FOR EXAMINATION D

DIFFERENTIAL EQUATIONS

I. Ordinary Differential Equations:

Existence and uniqueness of solutions of systems of linear and nonlinear ordinary differential equations, successive approximations, explicit solutions of constant coefficient linear systems including matrix exponentials, variation of parameters method for solving non-homogeneous systems, stability, oscillation, continuous dependence of solutions.

II. Partial Differential Equations:

Linear, quasi-linear, and nonlinear first-order equations; the classical mathematical models for the vibrating string, heat conduction and gravitational potential; characteristics, classification and canonical forms for second-order equations; the Cauchy problem and the Cauchy- Kovalevsky theorem; Holmgren's uniqueness theorem. Basic results for elliptic, parabolic and hyperbolic linear equations in one and several space dimensions; systems of linear and quasi- linear equations.

References:

I. Coddington and Levinson, Theory of Ordinary Differential Equations
Jordan and Smith, Nonlinear Ordinary Differential Equations

Waltman, A Second Course in Ordinary Differential Equations

II. John, Partial Differential Equations
Weinberger, A First Course in Partial Differential Equations

SYLLABUS FOR EXAMINATION E

NUMERICAL MATHEMATICS

Numerical Analysis

Floating-point arithmetic, roundoff errors, condition and stability. Interpolation by polynomials: Newton form, Lagrange form, divided differences, error formula, Chebyshev polynomials and uniform approximation, Chebyshev points and their use as interpolation nodes. Solution of nonlinear equations: bisection method, Newton's method, fixed-point iteration, convergence criteria, rates of convergence, Aitken's method. Systems of linear equations: Gaussian elimination, pivoting strategies, LU and PLU factorizations, vector norms, matrix norms condition numbers, iterative improvement. Least-squares approximation: normal equations, orthogonal polynomials and their properties. Numerical Integration: derivation of basic rules and their error formulas, Gaussian quadrature, composite rules, adaptive quadrature, deriving extrapolation formulas, Romberg integration. Numerical solution of initial-value problems for ordinary differential equations: local truncation error and global error, Taylor series methods, Runge-Kutta methods, derivations of multistep methods and their local truncation errors, implicit and explicit methods, predictor-corrector methods.

Numerical Linear Algebra

Basic Algorithms (saxpy, dot product, matrix multiplication, etc.). Vector norms and matrix norms. The inverse of a perturbation of a nonsingular matrix. Finite precision matrix computations. Forward and backward error analysis. Orthogonal matrices. The singular value decomposition: existence and basic properties. Sensitivity of square linear systems, condition number, linear systems of equations: Gauss transformations, Gaussian elimination, LU
factorization, round off analysis for Gaussian elimination, pivoting strategies, growth factor, residuals and errors, iterative improvement of solutions, condition estimation, banded systems, symmetric systems, diagonal dominance, positive-definite matrices, Cholesky factorization. Householder and Givens transformations: uses and algorithms. QR factorization algorithms (Householder, Givens, Gram-Schmidt and modifed Gram-Schmidt procedures). The full rank least-squares problem: sensitivity, algorithms, relative advantages and disadvantages of algorithms based on the normal equations, modifed Gram-Schmidt, and Householder QR. The unsym-metric eigenproblem: basic theory, Schur decomposition, invariant subspaces, algebraic and geometric multiplicity of eigenvalues, defective matrices, normal matrices, perturbation results (Gershgorin Circle Theorem, Bauer-Fike Theorem, the condition of a simple eigenvalue and the corresponding eigenvector). The power iteration, inverse iteration, orthogonal iteration and the basic QR algorithm. Hes-senberg matrices: reduction algorithms, basic properties; their role in the QR algorithm. Shifts for the QR algorithm: implicit and explicit, expected quadratic convergence. Real Schur form, Francis' QR algorithm (implicit double shifts in real arithmetic). Implementation of the QR algorithm for unsymmetric matrices. The symmetric eigenvalue problem: perturbation results (Wielandt-Hoffman Theorem), tridiagonal matrices; Wilkinson shifts; convergence result; implementation of the tridiagonal QR algorithm.

Numerical Differential Equations

Initial value problems for ODE (Euler, multistep, backward difference and Runge-Kutta methods). Finite difference methods for elliptic PDE (numerical differentiation, discretization of elliptic operators, existence and uniqueness of solutions to the discrete problem, error estimates in the norm and norm for the solution of Poisson's equation). Iterative methods for solving systems of linear equations (Gauss-Seidel, Jacobi, steepest descent, conjugate gradient). Finite elements for elliptic PDE (variational formulation of elliptic BVP's, Galerkin approximation, changes of variable, trial functions of degree higher than 1, algorithm for-constructing the discrete problem, structure of the stiffness matrix). Evolution equations (semidiscretization, parabolic equations using finite differences and finite elements, wave problem, transport equation, upwinding, Fourier stability method, shooting method).

References:

Atkinson, An Introduction to Numerical Analysis
Burden and Faires, Numerical Analysis
Ciarlet, Numerical Analysis of the Finite Element Method
Conte and de Boor, Elementary Numerical Analysis
Golub and Van Loan, Matrix Computations
Hall and Porsching, Numerical Analysis of Partial Differential Equations
Sewell, The Numerical Solution of Ordinary and Partial Differential Equations
Stewart, Introduction to Matrix Computations

SYLLABUS FOR EXAMINATION F

MATHEMATICS EDUCATION: FOUNDATIONS AND RESEARCH

This is a take-home examination. Preparation for this examination requires the exploration of and reflection on a range or topics and issues related to mathematics education.

Graduate students are expected to have knowledge of individuals, groups, and organizations whose work has contributed to current understandings and perspectives of the learning and teaching of mathematics, mathematics curriculum and assessment, and mathematics education research. This includes, but is not limited to, the work of Bruner, Dewey, Gagne, Montessori, Piaget, Skinner, Thorndike, Vygotsky, plus Begle, Brownell, Dienes, Romberg, Skemp, Steffe, von Glasersfeld, as well as the work of projects, e.g., SMSG and DMP, and organizations such as NCTM, NCSM, and MAA.

Graduate students' preparation should also include an examination of the history of reform and change in mathematics education over the last 50 years. Along with this, graduate students should have an awareness of the effects of various reform movements on current trends in classroom practices, perspectives, and research paradigms in mathematics education, both in general and as they relate to particular contexts (e.g., the teaching of algebra or geometry).

Graduate students are expected to have had experience reading and evaluating original research, and to have developed an awareness of and an appreciation for various research methods and models in mathematics education. In preparing for this examination, graduate students are expected to analyze and synthesize research results in developing a broad perspective of quality research. Graduate students are also expected to have knowledge of other issues concerned with planning, conducting and evaluating research, and implementing and extending the existing body of research knowledge.

Suggested References and Reading:

Barlage, E, (1982). The New Math, Ah Historical Account of the Reform of Mathematics Instruction in the United States of America. (ERIC Document [ED] Number 224703).

Bell-Gredler, M. E. (1986). Learning and Instruction: Theory into Practice. New York: Macmillan Publishing Company.

Breault, R. A. (1991). Educational Reform since 1945 and its Implications for Teacher Education. (ED 341679).

Cook, T. D. & Campbell, D. T. (1979). Quasi-experimentation: Design and analysis issues for field settings. Boston: Houghton Miffin.

Fennema E., Carpenter T. P., & Lamon, S. (Eds.) (1991). Integrating Research on Teaching and Learning Mathematics. Albany, NY: State University of New York Press.

Grouws, D. A. (Ed.) (1992). Handbook of research on mathematics teaching and learning. New York: Macmillan Publishing Company.

Grouws, D. A., & Cooney, T. J. (Eds.) (1988). Effective Mathematics Teaching. Reston, VA: NCTM, &; Lawrence Erlbaum Associates.

Hiebert, J. (Ed.) (1986). Conceptual and Procedural Knowledge: the case of mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates.

Kennedy, J. J. & Bush, A. J. (1985). An introduction to design and analysis of experiments in behavioral research. Lanham, MD: University Press of America.

Kirk, J. & Miller M. (1986). Reliability and Validity in Qualitative Research. Newbury Park, CA: Sage University.

Lapointe, A. (1989). A World of Differences: An International Assessment of Mathematics and Science. Princeton, NJ: Educational Testing Service.

The National Assessment of Educational Progress results in Mathematics (NAEP).

National Council of Teachers of Mathematics [NCTM] (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: Author.

NCTM (1991). Professional Standards for Teaching Mathematics. Reston, VA: Author.

NCTM (1995). Assessment Standards for School Mathematics. Reston, VA: Author.

Resnick, L. EL, & Ford, W. W. (1981). The psychology of mathematics for instruction. Hillsdale, NJ: Lawrence Erlbaum Associates.

Skemp, R, R. (1987). The Psychology of Learning Mathematics (Expanded American Edition). Hillsdale, NJ: Lawrence Erlbaum Associates.

Spradley, J. P. (1980). Participant Observation. New York: Holt, Rinehart & Winston.

Webb, N. L. & Romberg, T. A. (Eds.) (1994). Reforming mathematics education in America's cities: the urban mathematics collaborative project. New York: Teachers College Press.

Wittrock, M. C. (Ed.) (1986). Handbook of research on teaching. New York: Macmillan Publishing Company.

See also various other publications from the NCTM including yearbooks, monographs, and journals, i.e., Arithmetic Teacher, Teaching Children Mathematics, Mathematics Teaching in the Middle School, Mathematics Teacher, and Journal for Research in Mathematics Education. In addition, other relevant publications may include articles from journals such as the following: American Educational Research Journal Educational and Psychological Measurement Educational Researcher Educational Studies in Mathematics Focus on Learning Problems in Mathematics For the Learning of Mathematics Journal of Educational Psychology Journal of Educational Research Journal of
Mathematical Behavior Review of Educational Research School Science and Mathematics.

Suggested Coursework: MATH 610, MATH 611, and at least TWO other Graduate-level mathematics education courses at the 600-level (such as MATH 602, MATH 612, MATH 613, MATH 614, or MATH 615), or at the 700-level (such as MATH 710A, or MATH 710B).

SYLLABUS FOR EXAMINATION G

MATHEMATICS EDUCATION: THEORIES OF LEARNING & TEACHING

This is a three-hour written examination on the theories of learning and teaching of mathematics. Research-based literacy on students' and teachers' understandings of specific mathematical concepts is expected. Literacy with specific original research published in theses, dissertations, books, and periodicals is expected. Further, the graduate student is expected to link her/his knowledge about students' mathematical thinking and knowledge-building with how classroom instruction may be guided to enhance meaningful learning. Emphasis will be placed on the graduate student's in-depth analysis, synthesis and ability to extend the body of published research on the meaningful understanding and teaching of specific mathematical concepts and processes, at least, at any two of the following levels of education: elementary school, middle school, secondary school, and college level.

Suggested References:

Carpenter, F., E. Fennema, & T. Romberg (Eds.) (1993). Rational numbers: An integration of research. Hillsdale, N.J.: Lawrence Erlbaum.

Fennema, E., Carpenter, T. & Lamon, S. (Eds.) (1991). Integrating research on teaching and learning mathematics. New York: SUNY press.

Harel, G. & Confrey, J. (1994). The development of multiplicative thinking. New York: SUNY Press.

Hart, K, (Ed.) (1981). Children's Understanding of Mathematics:ll-16. London: John Murray.

Hiebert, J., & Behr, M. (Eds.) (1988). Number concepts and operations in the middle grades, Reston, VA.: National Council of Teachers of Mathematics, & Lawrence Erlbaum Associates.

Hiebert, J. (Ed.) (1986). Conceptual and procedural knowledge: The case of mathematics. Hillsdale, N.J.: Lawrence Erlbaum.

JRME Research Monographs.

Kamii, C, and Livingston, S.J. (1995). Young children continue to reinvent arithmetic, 3rd grade/ Implications of Piaget's theory. New York, NY: Teachers College Press, Columbia University.

Leinhardt, G., Putnam, R., Hattrup, R. (Eds.) (1992). Analysis of arithmetic for mathematics teaching. Hillsdale, N.J.: Lawrence Erlbaum.

Lindquist, M., & Schulte, A. (Eds.) (1987). Learning and teaching geometry, K-12: 1987 Yearbook. Reston, VA.: National Council of Teachers of Mathematics

Payne, J. (Ed.) (1990). Mathematics for the young child. Reston, VA.: NCTM.

Piaget (A selection of his publications)

Romberg, T., E. Fennema, & T. Carpenter (Eds.) (1993). Integrating research on the graphical representation of functions. Hillsdale, N.J.: Lawrence Erlbaum.

Schoenfeld, A. (Ed.) (1994). Mathematical thinking and Problem Solving. Hillsdale, N.J.: Lawrence Erlbaum.

Silver (Ed.) (1985). Teaching and learning mathematical problem solving: Multiple perspectives. Hillsdale, N.J.: Lawrence Erlbaum,

Steffe, L. (Ed.) (1991). Epistemological foundations of mathematical experience. New York: Springer-Verlag.

Steffe, L. & T. Wood (Eds.) (1990). Transforming children's mathematical experience. Hillsdale, N.J.: Lawrence Erlbaum.

Tsudura, G. (1994). Putting it together. Portsmouth, NH : Heinemann.

Wagner, S. and Kieran, C. (Eds.) (1989). Research issues in the learning and teaching of algebra: Vol 4. Reston, VA: NCTM.

Wittrock, M. (Ed.) (1986). Handbook of Research on Teaching. New York: McMillan..

Literacy with specific original research studies published in theses, dissertations, and research periodicals. A Selection of such periodicals is given below:

Focus on Learning Problems in Mathematics, Educational Studies in Mathematics, Journal for Research in Mathematics Education, For the Learning of Mathematics, Journal of Mathematical Behavior, Cognition k, Instruction.

Suggested Coursework: MATH 513, MATH 514, and at least TWO other graduate-level mathematics education courses at the 600-level (such as MATH 602, MATH 610, MATH 611, MATH 612, or MATH 615), or at the 700-level (such as MATH 710A, or MATH 710B).

SYLLABUS FOR EXAMINATION H

STATISTICS: PROBABLITY AND THEORY OF STATISTICS

I. Probability: probability spaces; measures; measurable functions and algebra of events; random variables,; expectations; characteristic and moment generating function;, Discrete, continuous, mixed and multivariate probability distributions; sequences of random variables and various modes of convergence; Borel-Cantelli Lemma and 0-1 laws; weak and strong law of large numbers; convergence in distributions and central limit theorems; conditional expectations and conditions distributions. Additional topics vary depending on the coverage in STAT 670 and may include martingales, Brownian motion and other stochastic processes, infinitely divisible and stable distributions, asymptotics, and various probability inequalities.

II. Theory of Statistics: exponential families; location and scale families; hierarchical models and mixture distributions; sampling distributions; properties of sample mean and variance from normal distribution,; sufficiency principle; complete families; point estimation including unbiasedness, maximum likelihood and Bayesian estimation; consistency; hypothesis testing and interval estimation. Additional topics vary depending on the coverage in STAT 672 and may include statistical decision theory, asymptotics, and higher-order theory.

References:

I.

1. Gut. A. (2005), Probability: A Graduate Course. Springer-Verlag, New York.

2. Pollard, D. (2002), A User's Guide to Measure Theoretic Probability, Cambridge University Press.

3. Shorack, G.R. (2000), Probability for Statisticians, Springer-Verlag, , New York.

II.

1. Casella, G. and Berger, R. (2001), Statistical Inference, Duxbury.

2. Mukhopadhyay, N. (2000), Probability and Statistical Inference, Marcel-Dekker.

3. Schervish, M. (1995), Theory of Statistics, Springer-Verlag, New York.

4. Young, G.A. and Smith, R.L. (2005), Essentials of Statistical Inference, Cambridge University Press.

SYLLABUS FOR EXAMINATION I

STATISTICS: LINEAR MODELS AND BAYESIAN STATISTICS

I. Linear Models: Multivariate normal distribution; distribution of quadratic forms; linear models and design matrix of less than full rank; estimation and distribution theory; generalized least squares; hypothesis testing and distribution theory for F-test; confidence interval and regions; multiple comparisons; analysis of variance. Additional topics vary depending on the coverage in STAT 673 and may include polynomial regression, departure from assumptions and diagnostics, prediction, and model selection.

II. Bayesian Statistics: Bayesian inference,; loss function and risk; one parameter models and posterior inference; conjugate priors; non-informative priors; multi-parameter models; Bayesian computation,; Gibbs sampling; Markov chain Monte Carlo methods and applications in different areas. Additional topics may include decision theory, theoretical and convergence properties of Markov chain samplers, Bayesian model checking, model selection and assessment criteria, hierarchical models and Bayesian survival analysis.

References:

I.

1. Christensen, R. (2002), Plane answers to complex questions, 3 rd edition, Springer-Verlag/

2. Seber G.A.F. and Lee J.A. (2003), Linear Regression Analysis, 2nd Ed., Wiley.

3. Ravishankar, N. and Dey, D.K. (2001), A First Course in Linear Model Theory, Chapman and Hall/CRC.

4. Rencher, A. C. and Schaalje, G.B. (2008), Linear Models in Statistics, 2nd Ed, Wiley.

II.

1. Robert, C.P. (2007), The Bayesian Choice From Decision-Theoretic Foundations to Computational Implementation, 2nd Ed., Springer-Verlag, New York.

2. Christensen, R., Johnson, W., Branscum, A., and Hanson, T. (2010), Bayesian Ideas and Data Analysis: An Introduction for Scientists and Statisticians, Chapman & Hall/ CRC Press, Boca Raton.

3. Carlin B.P., Louis TA (1996), Bayesian Methods for Data Analysis, 3 rd Ed., Chapman & Hall/ CRC Press, London.

4. Berger J.O. (1980), Statistical Decision Theory: Foundations, Concepts, and Methods. Springer – Verlag, New York, Springer Series in Statistics.

5. Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. (2003), Bayesian Data Analysis, second ed., CRC Press.

SYLLABUS FOR EXAMINATION J

STATISTICS: ADVANCED STATISTICAL METHODS AND STATISTICAL CONSULTING

Exam J is a take-home exam. The student will have to turn-in the completed exam within 7 days of when the exam is given to the student. Exam J emphasizes methodologies, real data analysis, implementation in software, professional quality reporting, and *engaged learning*.

Statistics, as a subject and discipline, covers a spectrum with statistical methods in the middle and theory and applications on the two sides. The goal of Exam J is to judge the student's expertise in methods and applications and the student's ability to transition (in either direction) between methods and applications.

Exam J, with its two components, will ask the student(s) to answer specific questions and will not be an open-ended research project. One goal of Exam J is to evaluate the student's skill-set for statistical methods and applications. These skills may become necessary for the student in future collaborative (and possibly interdisciplinary) research projects, though this not the only purpose Exam J is designed to serve. Exam J will follow the standard setting of an examination where the answers will be judged against (at least one set of) established solutions (or approaches or methods).

I. Advanced Statistical Methods: Varied topics on recent statistical methodologies and applications. Topics vary depending on the coverage in STAT 679 and may include generalized linear models, linear mixed models, generalized linear mixed models, statistical methods for modeling and analyzing longitudinal data, methods for analyzing missing data, resampling methods, and multivariate and categorical data analysis, statistical data mining, analysis of high dimensional data, and statistical bioinformatics.

This part of the exam will carry 50% weight to the overall score for Exam J.

This part of the exam may include theory and methodological questions where the students are asked to make methodological developments and possibly establish theoretical properties of the method(s). The student may also be asked to modify a method appropriately so that it fits the need of a specific application, establish the properties of the modified method, and then apply to the modified method to the application.

II. Statistical Consulting: Topics vary depending on the coverage in STAT 691 and may include techniques for problem formulation; identification of parameters and solutions; ill-posed problems and their formulation.

Note that the Statistical Consulting course is an engaged learning course where the students use statistical methodologies in real world problems.

This part of the exam will carry 50% weight to the overall score for Exam J.

This part of the exam will focus on analysis of real data. The Exam may utilize data from the following sources and others.

1. Data which are already published in statistical and other scientific literature. In this case, the analysis reported in the publication may serve as a standard or baseline against which the student's answer will be judged.

2. Data which were brought in to the Statistical Consulting Center (SCS) by a client, for which the client provides written permission for use in the Qualifying exam and whose analysis the instructor of STAT 691 deems complex enough to be asked in Qualifying Exam J. In this case, the statistical analysis already done by SCS may serve as a standard or baseline against which the student's answer will be judged.

3. Data from other sources. For example, Sanjib, while teaching STAT 691, used an extensive dataset (under permission from the study investigators to use these data in the course) on longitudinal measurements from 902 heart patients in a clinical trial. In this case, the statistical analysis already done on the data may serve as a standard or baseline against which the student's answer will be judged.

The goals of this part of the exam are the following:

1. to judge the student's readiness, skill-set and expertise in handling real data;

2. to judge the student's readiness, skill-set and expertise in formulating scientific question(s) into statistical question(s);

3. to judge the student's readiness, skill-set and expertise in formulating a statistical model, in choosing appropriate statistical methodologies and in modifying statistical methodologies to meet the intricacies of the data.;

4. to judge the student's readiness, skill-set and expertise in implementation, in developing appropriate code and software for implementing the modified methodologies.;

5. to judge the student's readiness, skill-set and expertise in appropriate presentation of the results from the statistical analysis in a readily interpretable and client-quality professional report.

As mentioned before, the students will be asked to answer specific scientific questions and the answers will be judged against established standards/solutions/approaches. The setting will be that of a take-home exam rather than an open-ended consultancy or research project.

References:

I and II.

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