

“None” means “None of the previous answers is correct.”

Question 1. Which of the following two regions contains the point $(-3, 2)$:

$$\text{I. } y \leq x \qquad \text{II. } y - 3x \leq -4$$

(a) Neither I nor II (b) I only (c) II only (d) both I and II (e) None

Question 2. True or False

I. Any column matrix A can be multiplied by any row matrix B in the order AB .

II. When reducing the matrix $[A|B]$, corresponding to a linear system of 4 equations in 5 variables, if the column matrix B consists of all zeros, then the system has infinitely many solutions.

(a) both are true (b) only I is true (c) only II is true (d) both are false (e) None

Question 3. The matrix $\begin{bmatrix} 3 & x \\ 4 & 36 \end{bmatrix}$ has no inverse if x is

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) 27 (e) None

Question 4. Mr. Jones has \$9000 to invest in three types of stocks: low-risk, medium risk, and high-risk. He invests according to three principles. The amount invested in low-risk stocks will be at most \$1000 more than the amount invested in medium-risk stocks. At least \$5000 will be invested in low- and medium risk stocks. No more than \$7000 will be invested in medium- and high-risk stocks. The expected yields are 6% for low-risk stocks, 7% for medium-risk stocks, and 8% for high-risk stocks. If x is the amount of money Mr. Jones invests in medium-risk stocks and y is the amount of money he invests in high-risk stocks, what is the objective function for his total yield, which he wants to maximize?

(a) $720 - .02x - .01y$ (b) $630 - .01x + .01y$ (c) $540 + .01x + .02y$ (d) $9000 - .06x - .07y$
(e) None

Question 5. If $A = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, find $2B - A^{-1}$.

- (a) $\begin{bmatrix} -3 & 2 \\ -13 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 10 \\ -3 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ -13 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (e) None

Question 6. The solution X for the matrix equation $AX = D$ is

- (a) $X = (I - A)^{-1}D$ (b) $X = (A - I)^{-1}D$ (c) $X = A^{-1}(I - D)$ (d) $X = A^{-1}D$
(e) $X = DA^{-1}$

Question 7. True or false.

I. If there are no negative values in the bottom row of a Simplex tableau *in standard form*, then the problem is finished.

II. For a linear programming problem, if the maximum of P occurs at (x, y) , then the minimum of $M = -P$ occurs at $(-x, -y)$.

- (a) both are true (b) I only (c) II only (d) neither is true (e) None

Question 8. Find the maximum of $P = 5x + 10y$ subject to the constraints:

$$x + y \leq 150$$

$$2x + y \leq 200$$

$$x \geq 10$$

$$y \geq 20$$

- (a) 1850 (b) 1450 (c) 850 (d) 650 (e) None

Question 9. For a linear programming problem in two variables, the objective function is to be maximized. Which of the following situations are *possible*?

I. The maximum occurs at exactly two points of the feasible set.

II. The feasible set is the empty set.

III. The objective function does not have a maximum on the feasible set.

- (a) I only (b) I and II only (c) I and III only (d) all three are possible (e) None

Question 10. A farmer has 100 acres of land on which to grow tomatoes and peas. The cost of planting each acre of tomatoes is \$ 50. Each acre of peas will cost \$ 25 to plant. The farmer's budget for planting is \$ 6250. If the anticipated profits are \$ 100 per acre of peas and \$ 150 per acre of tomatoes, how many acres of each should be planted to maximize profits? If x denotes the number of acres of tomatoes to be planted and y the number of acres of peas, then the profit to be maximized is:

- (a) $P = 50x + 25y$ (b) $P = 150x + 100y$ (c) $P = 100x + 150y$ (d) $P = 25x + 50y$
 (e) None

Question 11. What is the next pivot element for the Simplex tableau

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 40 \\ -1 & 5 & 0 & 1 & 0 & -15 \\ 3 & -4 & 0 & 0 & 1 & 0 \end{bmatrix} ?$$

- (a) -1 (b) 1 in row 1, col. 2 (c) 2 (d) 5 (e) None

Question 12. Read the *current* solution (x, y, z) from the following tableau:

$$\begin{bmatrix} x & y & z & u & v & w & M & \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 & 0 & 0 & 0 & 8 \\ 1 & 1 & 0 & 4 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & -3 & -6 & 0 & 1 & 20 \end{bmatrix}$$

- (a) (4, 8, 3) (b) (3, 0, 8) (c) (3, 4, 8) (d) (3, 3, 8) (e) None

Question 13. Minimize $C = 6x + 4y + 2z$ subject to the constraints:

$$\begin{aligned} x + 2y - 5z &\geq 15 \\ x, y, z &\geq 0 \end{aligned}$$

The minimum occurs when y equals

- (a) 0 (b) 7.5 (c) 15 (d) 30 (e) None

Question 14. How many subsets does the set $\{1, 2, 3\}$ have?

- (a) 3 (b) 4 (c) 6 (d) 8 (e) None

Question 15. A total of 37,451 Ph.D.'s were earned in 1991. Out of the 1164 Ph.D.'s in business and management, 292 were earned by women. Women earned a total of 13,782 Ph.D.'s. How many men earned Ph.D.'s in business and management?

- (a) 13,490 (b) 872 (c) 23,669 (d) The number cannot be determined. (e) None

Question 16. A merchant surveyed 400 people to determine the way they learned about an upcoming sale. The survey showed that 180 learned about the sale from the radio, 185 learned about the sale from TV, and 180 learned about the sale from the newspaper. Fifty people learned about the sale from the radio only, 60 people from TV only, and 70 from the newspaper only. In addition, 100 people learned about the sale from radio and TV. How many people sampled learned of the sale from all three sources?

- (a) 30 (b) 45 (c) 55 (d) 100 (e) None

Question 17. If $A = \{a, d, g, j\}$ and $B = \{a, b, c, d, p, q, r\}$ then $A \cup B =$

- (a) $\{b, c, g, j, p, q, r\}$
(b) $\{a, b, c, d, g, j, p, q, r\}$
(c) $\{a, d\}$
(d) B
(e) None

Question 18. How many different ways can a 25-member football team select a captain and an assistant captain?

- (a) $25 \cdot 24$ (b) $25!$ (c) $\frac{25 \cdot 24}{2}$ (d) 25^2 (e) None

Question 19. Calculate $\frac{(n-2)!}{n-2}$.

- (a) 1 (b) $n-1$ (c) $(n-1)!$ (d) $(n-3)!$ (e) None

Question 20. An urn contains 8 green and 6 purple balls. How many ways can a sample of 3 balls contain *at least one* purple ball?

- (a) 56 (b) 140 (c) 288 (d) 308 (e) None

Question 21. An experiment consists of tossing a coin four times and observing the sequence of heads and tails. Compute $\Pr(E)$, where E is the event that more heads occur than tails.

- (a) $1/2$ (b) $5/16$ (c) $3/8$ (d) $7/16$ (e) None

Question 22. The probabilities that two species become extinct in 5 years are .3 and .2 respectively. Assuming that these probabilities are independent, what is the probability that at least one of the species will become extinct in the next 5 years?

- (a) .44 (b) .5 (c) .6 (d) .06 (e) None

Question 23. Which of the following are *independent* events?

I. E and F if $\Pr(E) = .6$, $\Pr(F) = .5$, and $\Pr(E | F) = .3$

II. A and B if A and B are represented by the Venn Diagram

- (a) Both I and II (b) I only (c) II only (d) neither (e) None

Question 24. A basketball player with a free-throw shooting average of .6 is on the line to shoot *three* free throws. Assuming that the three throws are independent, what is the probability that the player will score exactly 1 point?

- (a) .24 (b) .288 (c) .48 (d) .60 (e) None

Question 25. Assume that following data: 20% of all Americans smoke, 5% of all smokers develop lung cancer; while 0.1% of all nonsmokers develop lung cancer. If a person has lung cancer, what is the probability that he or she smokes?

- (a) $5/9$ (b) $25/27$ (c) $50/51$ (d) $125/126$ (e) None

Question 26. Fran buys three kinds of cat food: canned, packaged, and dry. Her cat likes variety, so she never serves the same kind on successive days. If she serves canned one day, then the next day she serves packaged, but if she serves packaged or dry, then the next day she is half as likely to serve canned as the other types. The transition matrix is:

$$(a) \begin{array}{c} \text{C} \\ \text{P} \\ \text{D} \end{array} \begin{bmatrix} \text{C} & \text{P} & \text{D} \\ 0 & \frac{2}{3} & \frac{2}{3} \\ 1 & 0 & \frac{1}{3} \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$$

$$(b) \begin{array}{c} \text{C} \\ \text{P} \\ \text{D} \end{array} \begin{bmatrix} \text{C} & \text{P} & \text{D} \\ 1 & \frac{2}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$(c) \begin{array}{c} \text{C} \\ \text{P} \\ \text{D} \end{array} \begin{bmatrix} \text{C} & \text{P} & \text{D} \\ 0 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \\ 1 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$(d) \begin{array}{c} \text{C} \\ \text{P} \\ \text{D} \end{array} \begin{bmatrix} \text{C} & \text{P} & \text{D} \\ 0 & \frac{1}{3} & \frac{1}{3} \\ 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

(e) None

The next two problems refer to the following information:

A math professor gives pop quizzes in a class that meets five days a week (Monday thru Friday). If he gives a pop quiz on one day, he will *not* give a quiz the next day; however, if he doesn't give a pop quiz one day, there is a $\frac{2}{3}$ chance he will give a pop quiz the next day.

Question 27. What is the probability there will be a quiz on Thursday, given that there is not a quiz on Tuesday?

- (a) 0 (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$ (e) None

Question 28. In the long run, what is the probability there will *not* be a quiz on any particular day?

- (a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$ (e) None

Question 29. The matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

is

- (a) non-stochastic
- (b) stochastic and regular
- (c) stochastic and absorbing
- (d) stochastic and non-absorbing
- (e) None

Question 30. The stable matrix of the absorbing stochastic matrix $\frac{\text{I} \mid \text{S}}{0 \mid \text{R}}$ is

- (a) $\frac{\text{I} \mid S(I - R)^{-1}}{0 \mid 0}$
- (b) $\frac{\text{I} \mid R(I - S)^{-1}}{0 \mid 0}$
- (c) $\frac{\text{I} \mid 0}{0 \mid S(I - R)^{-1}}$
- (d) $\frac{\text{I} \mid 0}{0 \mid R(I - S)^{-1}}$
- (e) None