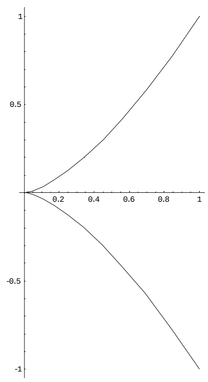
1. **(a)** Since $x(t) = t^2$ and $y(t) = t^3$, it must be that $t = -2$ at the point $(4, -8)$. The slope is given by $\dfrac{dy/dt}{dx/dt} = \dfrac{3t^2}{2t} = \dfrac{3t}{2}$ so when $t = -2$ it is $m = -3$. The tangent line is $y = -3x + b$ and $-8 = -3 \cdot 4 + b$ implies $b = 4$ so $y = -3x + 4$.

(b)

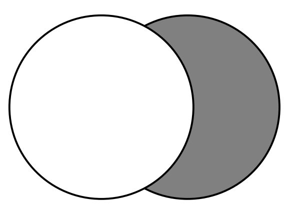


2. **(a)** $r = 1$

(b) $(x-1)^2 + y^2 = 1$ is a circle or radius 1 with center at $(1, 0)$. In polar form it is $(r\cos\theta - 1)^2 + r^2\sin^2\theta = 1$ or $r^2\cos^2\theta - 2r\cos\theta + 1 + r^2\sin^2\theta = 1$. This simplifies to $r = 2\cos\theta$.

(c) The intersections of the two curves occur when $2\cos\theta = 1$, so $\theta = \pm\pi/3$.

(d)

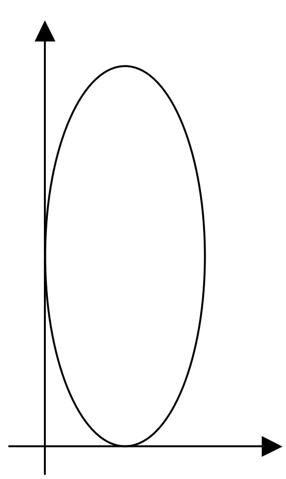


(e) $\frac{1}{2}\int_{-\pi/3}^{\pi/3}\left((2\cos\theta)^2 - 1^2\right)d\theta$

(f)

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3}\left(4\cos^2\theta - 1\right)d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}(2\cos 2\theta + 2 - 1)\,d\theta = \frac{1}{2}[\sin 2\theta + \theta]_{-\pi/3}^{\pi/3} = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$$

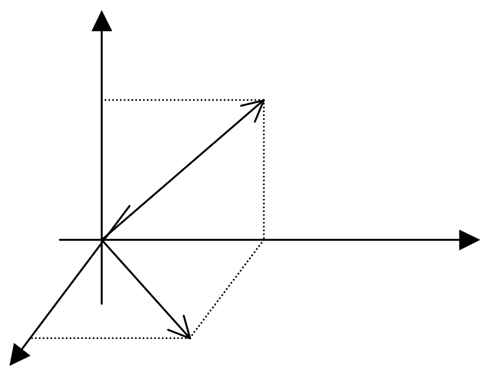
3. **(a)**



(b) The center is at $(3,5)$ so the equation is $\dfrac{(x-3)^2}{9}+\dfrac{(y-5)^2}{25}=1$.

(c) The foci are on the major axis and $c^2+3^2=5^2$ so $c=4$ offset from the center. The coordinates are $(3,1)$ and $(3,9)$.

4. (a)



(b) $\cos\theta=\dfrac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}=\dfrac{<1,1,0>\cdot<0,1,1>}{|<1,1,0>||<0,1,1>|}=\dfrac{1}{\sqrt{2}\cdot\sqrt{2}}=\dfrac{1}{2}$ so $\theta=\pi/3$

5. The projection of $\vec{v}$ onto $\vec{w}$ is given by $(\vec{v}\cdot\vec{u})\vec{u}$ where $\vec{u}$ is a unit vector in the direction of $\vec{w}$. Here $|\vec{w}|=|<3,0,-4>|=\sqrt{3^2+0^2+(-4)^2}=5$ so $\vec{u}=\dfrac{1}{5}<3,0,-4>$, and $(\vec{v}\cdot\vec{u})\vec{u}=(<8,3,1>\cdot\dfrac{1}{5}<3,0,-4>)\dfrac{1}{5}<3,0,-4>=\dfrac{24-4}{25}<3,0,-4>=\dfrac{4}{5}<3,0,-4>$.

6. **(a)** The distance from the line $\vec{p}+t\vec{v}$ to the origin is given by $|\vec{p}\times\vec{u}|$ where $\vec{u}$ is a unit vector in the direction of $\vec{v}$. Here $|\vec{v}|=|<-1,2,0>|=\sqrt{(-1)^2+2^2+0^2}=\sqrt{5}$ so $\vec{u}=\dfrac{1}{\sqrt{5}}<-1,2,0>$ and

$$\vec{p}\times\vec{u}=<1,0,0>\times\frac{1}{\sqrt{5}}<-1,2,0>=$$
$$\frac{1}{\sqrt{5}}<0\cdot0-0\cdot2,0\cdot(-1)-1\cdot0,1\cdot2-0\cdot(-1)>=\frac{1}{\sqrt{5}}<0,0,2>.$$

The magnitude is $2/\sqrt{5}$.

(b) With $y=-2x+2$, the distance squared has derivative $\left(x^2+(-2x+2)^2\right)'=2x+2(-2x+2)\cdot(-2)=10x-8$. The derivative is zero only if $x=4/5$ and the corresponding value is $(\frac{4}{5})^2+((-2)\frac{4}{5}+2)^2=\frac{16}{25}+\frac{4}{5}=\frac{4}{5}$. The square root of this is $2/\sqrt{5}$.

(c) This value is the same as in part (a) as is expected since the two lines $\vec{p}+t\vec{v}$ and $y=-2x+2$ are both in the xy-plane and are in fact the same line. The two ways of calculating the distance from this line to the origin must produce the same value.